Targeting in Multi-Class Screening Under Error and Error-Free Measurement System

by

Atiq Waliullah Siddiqui

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

SYSTEMS ENGINEERING

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This thesis, written by, ATIQ WALIULLAH SIDDIQUI under the supervision of his thesis advisor and approved by his thesis committee, has been presented to and accepted by the Dean of Graduate Studies, in partial fulfillment, of the requirements for the degree of MASTERS OF SCIENCE IN SYSTEMS ENGINEERING.

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عمادة الدراسات العليا
DEANSHIP OF GRADUATE STUDIES

To my dear parents

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All praise be to Allah (Subhanhu Wa T'aala). I am thankful to Allah (Subhanhu Wa T'aala) for his limitless help and guidance. peace and blessings be upon Muhammad (Sallallahu 'Alayhi Wassalam) the last of the messengers.

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خلاصة الأطروحة

الاســــم عتيق ولي الله صديقي

عنوان الأطروحة تهديف العمليات ذات منتج متعدد التصانيف تحت أنظمة قياس خالية وغير خالية من الخطاء

التخصص الدقيق هندسة نظم

التاريــخ يناير/٢٠٠١

لقد ازداد في الآونة الأخيرة اهتمام الباحثين باقتصاديات ضبط الجودة والتي يمكن تقسيمها إلى ثلاثة مجالات رئيسية هي : اقتصاديات خرائط التحكم، التصميم الاقتصادي لخطط الفحص والاختيار لبارامترات العمليات الصناعية، وهذا المجال الأخير يعني بتحديد القيم المثلى لبارامترات العملية الصناعية . يهدف هذا البحث لتطوير ثلاثة نماذج لتحديد القيمة المثلى لبارامترات العمليات الصناعية . النموذج الأول طُوّر للحالة التي يكون فيها عدة تصانيف للمنتج مع ووجود أخطاء في القياس . أظهرت النتائج الأثر الفعّال لاختيار متوسط العملية عند قيمته المثلى وفي النموذج الثاني تم إدخال مفهوم الانتظام في المنتجات وذلك بواسطة دالة الفقدان التربيعية (تاقوشي) عندما يكون نظام القياس دقيق وليس به أي مقدار من الخطأ . النتائج أوضحت زيادة ملوحظة في الربح عند استخدام النموذج الجديد . النموذج الثالث طُوّر بحيث يتضمن كلاً من حالتي الانتظام في المنتج وأخطاء القياس . أبانت النتائج أن النموذج أفضل من كل النماذج الأخرى فيما يترتب على استخدامه من فوائد عديدة . وقد تمت دراسة حساسية النماذج تجاه العديد من العوامل مثل أخطاء القياس وتكلفة الإنتاج. علاوة على ذلك تمت مقارنة النماذج تحليلياً ودراسة العلاقة بينها . كما تمت مقارنة حسابية للتعرف على الفائدة إذا تم استخدام نموذج أكثر واقعية . وختمت الرسالة بتقديم اقتراحات للبحوث المستقبلية .

ماجستير العلوم

جامعة الملك فهد للبترول والمعادن

الظهران، المملكة العربية السعودية

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THESIS ABSTRACT

FULL NAME ATIQ WALIULLAH SIDDIQUI

TITLE OF STUDY TARGETING IN MULTI-CLASS SCREENING UNDER ERROR AND ERROR-FREE MEASUREMENT SYSTEMS

MAJOR FIELD SYSTEMS ENGINEERING

DATE OF DEGREE JANUARY 2001

Recently, the economics of Quality Control has received a lot of interest from researchers. The Economics of Quality Control can be divided into three areas. These areas are "Economics of Control Charts", "Economic Design of Inspection Plans" and "Economic Selection of Process Parameters". The last problem is known as "Process Targeting". The objectives of the thesis were to develop three Process Targeting models. The first model was developed for the case of multi-class screening situation when the measurement system is considered error prone. The results showed nullifying effect of measurement error by optimally adjusting the process mean and the cut off points. In the second model, product uniformity was introduced via a Taguchi type quadratic loss function when the measurement system is considered error free. The results showed gains in profit in most of the cases if the new model is used. In the third model, a generalized model was presented in which both the product uniformity and the measurement error were incorporated. Results showed great deal of benefits over all other models if this integrated model is used. Sensitivity analysis has been performed to study the effect of measurement error and other parameters like production cost etc. Moreover, an analytic comparison of the models was performed and relationships between the models has been investigated. A numerical comparison has also been conducted to study the profit gain if a more realistic model is used. The thesis concluded by suggesting a number of extensions to be considered in future research.

MASTER OF SCIENCE DEGREE

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CHAPTER 1

INTRODUCTION

1.1 QUALITY CONTROL

In a manufacturing environment the product has to go across a number of processes undergoing diverse operations before obtaining a final form. Due to the natural or random, technological or assignable inconsistencies, especially systems of the nature of mechanical, chemical etc., it is bound to have some variations in the final product. In order to minimize this variation, and to improve the overall characteristics of the product, quality control became an essential part of manufacturing.

Most enterprises have started considering quality control as an integral part of their production process. The aspiration of improving quality came from the fact that major organizations failed to provide their customers with flawless products. This objective of total flawlessness is unattainable due to the inherent variability in the system, however it can be abridged and maintained at a certain level.

The definition of quality has evolved over the time. At first, it was defined as the "fitness for use". Another and more accurate, definition is that the "characteristics of a product should meet its specifications". The preceding definition, although more precise and objective than the first one, does not contemplate the deviation of the product characteristics within the specification limits. The foremost definition comes from the quality engineering that considers this deviation from a target value within the specification limit and imposes a penalty for it.

1.1.1 STATISTICAL QUALITY CONTROL

Statistical Quality control (SQC) is a subject that imparts a framework for obtaining enhancement in the quality of a product. SQC provides a number of ways to obtain this objective. These include "Product control" through 100% inspection or sampling inspection etc. and "Process control" by the use of control charts etc.

1.1.2 QUALITY ENGINEERING

As compared to SQC, this philosophy not only considers the quality at the final product or production stage but it also provides a new quality system in which the quality aspect is introduced right from the conceptual design to the final product phase. Another major difference in this philosophy is that it measures deviation of the quality characteristic from the target.

The most significant aspect of quality engineering, as opposed to the famous goal post philosophy (Philip) (see Figure 1-1), is its consideration of the deviation of the product from its target value

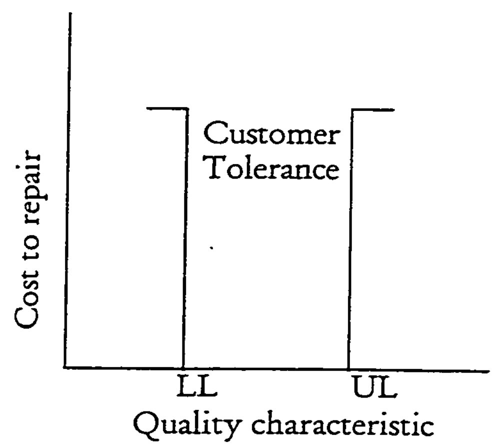


Figure 1-1: Goal post syndrome

within control limits. The loss due the product being off target is considered as a loss to the society. Taguchi presented a quadratic penalty for this deviation known as "Taguchi Quadratic Loss Function" that usually has the shape in Figure 1-2.

One of the major driving forces behind the research work done in the area of quality control is the economics associated with it. It may include the design of control charts or sampling plan not only from the quality outlook, but also from the considerations of cost/profit coupled to it. Another imperative area in economics of quality control is known as Process Targeting. The topic is discussed in detail in the next section.

1.2 PROCESS TARGETING

An important aspect of SQC is the determination of the optimum values (from economic standpoint) of the process parameters or machine settings/levels. The problem, generally known as "Process Targeting", relates the conformity of the product to the cost of production by finding certain settings of process parameters or machine settings/levels.

Due to inherent variability in the system the product may or may not be able, to fall above or at least on the minimum desired level. To increase the level of acceptance of the product, the process parameters may be set higher than their intended level, resulting in a cost of over doing (e.g., over filling the can etc.). Therefore, the general "Process Targeting" problem is to find the optimal settings of the process mean and other process parameters to minimize total cost resulting from quality cost and cost of manufacturing costs of material and processing etc.

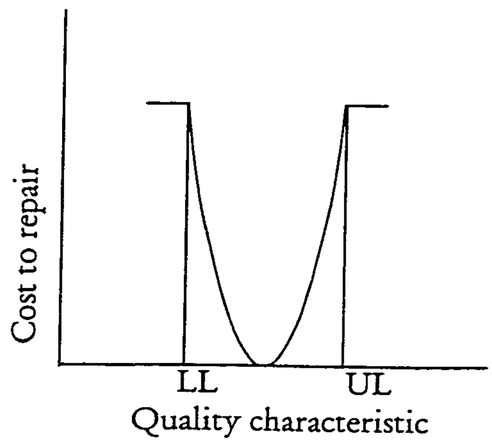


Figure 1-2: Taguchi loss function

The Initial "Process Targeting" problem has been presented in [1] and defined as follows:

A can filling process is considered. The quality characteristic/attribute is assumed to be the net weight of the filled can, (the weight of the can is assumed constant). The value of this attribute is a random variable Y. This attribute has lower (LSL) and artificial upper (USL) specification limits, which are known. The product is accepted if LSL$\leq$ Y $\leq$ USL and rejected otherwise. The process was assumed to be, 1) normally distributed with know variance, 2) Pearson type III. The inspection is assumed 100% and done by an automatic (assumed, error free) system. The objective is to minimize the expected cost.

The above problem has been extended and several models have been developed for these extensions. The models, presented in the literature are for the above problem with different assumptions and sampling plans. The assumptions include e.g., rejected part being reprocessed, measurement under error, machines in series etc. Despite a wide spectrum of the problems variation has been addressed, very few have considered the case where the product is screened in different grades (the process of classifying items in different grades is known as multi-class screening). Another important aspect that is not addressed very adequately in the literature is that the associated errors in inspection or measuring instruments.

1.3 MEASUREMENT ERROR

The classification of product off a production process, for quality control, is done by inspection (automatic or manual). This inspection requires some form of measurement in most of the cases.

There are always sources of error in a measurement system. The term accuracy and precision are often used in this connection. An accurate measurement system is the one that contains no systematic positive or negative errors about the true value, this is known as unbiased measurement. On the other hand, high precision means that the measurement will be made with little or no random variability or noise in the measured value (see figure 1-3). Inspection or measurement errors are active area of research as its impact on any quality control system is significant.

In this thesis, an attempt is made to study the impact of measurement error on process targeting and extend existing models in this area by incorporating Taguchi concepts in these models.

1.4 OBJECTIVES OF THE THESIS

The objectives of this thesis are to extend the targeting models for the multi class-screening situation by incorporating consistency criterion under error and error free measurement systems. Specifically the objectives are:

1. Develop a targeting model for multi class screening that incorporates the effects of measurement systems with error.

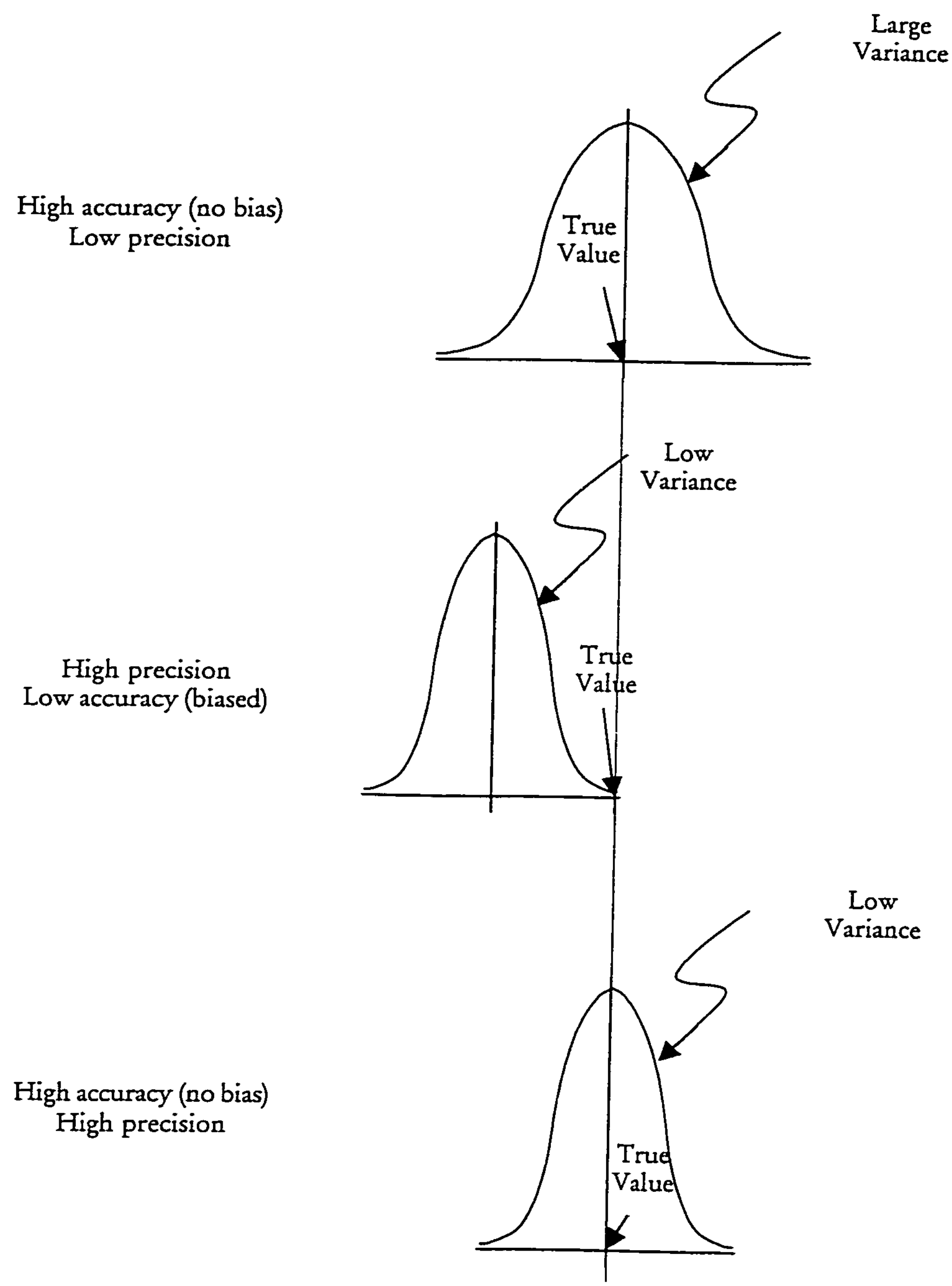


Figure 1-3: Biasness (accuracy) versus precision

2. Develop a multi class screening targeting model that incorporates the concept of Taguchi quadratic loss function under error free measurement system.
3. Extend the model resulting in objective 2 for the case of measurement systems with error.
4. Study the effect of error in the measurement system on the models that will be developed in objective 2 and 3 above.

1.5 THESIS ORGANIZATION

The notations used are common throughout the thesis and are described in the "Nomenclature" section. The notations that are needed for specific chapters will be presented in these chapters. A review of the literature of the area of "Process Targeting" is presented in Chapter 2. In the same chapter, the model of Min Koo Lee & Joong Soon Jang (1997) is described, since the work in this thesis extends this model. In chapter 3, the multi class-screening model with measurement error is presented followed by the model that incorporated Taguchi loss function in chapter 4. Chapter 5 contains a model for a multi class screening targeting that incorporates both Taguchi loss function and measurement error.

In chapter 6, the results of the comparative study between different models is discussed. Finally, conclusions and further research are outlined in chapter 7.

CHAPTER 2

LITERATURE REVIEW AND PROBLEM DEFINITION

2.1 INTRODUCTION

The purpose of this chapter is to present the literature review on the targeting problem. In addition, an outline of the basic model that is extended in this thesis is presented.

2.2 LITERATURE REVIEW

In this section, a brief literature review is given for the area of "Process Targeting". The literature is presented in chronological order.

C. Springer (1951) firstly considered the problem of optimal "Process Targeting". The problem was to determine the optimal mean for a canning process with specified upper and lower limits. He considered the price of producing under filled and over filled cans as fixed, but different.

D. Bettes (1962) proposed an empirical method, for the same model as in C. Springer (1951). His method was based on trial and error. The method is computationally tedious and does not give precise results.

W. Hunter and C. Kartha (1977) proposed a model in which under filled cans are sold in a secondary market at a reduced price. Cans above the lower specification limit are sold at a fixed price, which is an unrealistic assumption as cans barely over filled or nearly full are sold at a same price.

L. Nelson (1979) provided a nomograph for the model presented in C. Springer (1951).

An extension of W. Hunter and C. Kartha (1977) model was presented by **S. Bisgaard W. Hunter and L. Pallensen (1984)** where cans filled below specification limit are sold in a secondary market at a price proportional to the filled quantity.

D. Golhar (1987) studied the targeting problem with the assumption that under filled cans are to be emptied and refilled at the expense of a fixed reprocessing cost. The process is assumed to have known variance.

R. Vidal (1988) provided a simple graphical solution for the problem stated in S. Bisgaard W. Hunter and L. Pallensen (1984).

D. Golhar and S. Pollock (1988) extended the model presented by D. Golhar (1987) for the case where, the ingredient was assumed expensive. For that reason, upper specification limit of the quality characteristic, and the process mean, were determined.

The model reduces to the model presented in D. Golhar's [6] as upper limit tends to infinity.

M. A. Rahim and P. K. Banerjee (1988) firstly considered the process where the system has a linear drift (e.g., tool wear etc). A cost model for finished product is presented. A search algorithm as well as a graphical method is suggested to the optimal production run.

D. Golhar (1988) provided a computer program for the above model.

Another extension of the W. Hunter and C. Kartha (1977) is done by **Carlsson (1989)(a)**. In this model the producer gives compensation for the under filled item, along with a benefit for the producer for the over filled items. The compensation penalty is a fixed amount plus an amount proportional to the difference between the lower and upper limit.

O. Carlsson (1989)(b) determined, for the case of two variable characteristics, the optimum process mean under acceptance variable sampling.

R. Schmidt and P. Pfeifer (1989)(a) investigated the effects on cost savings from variance reduction in a single level canning problem and an approximate simple linear relationship between percentage reduction in standard deviation and the cost reduction was presented.

An extension of D. Golhar (1987) model for the case of capacitated filling is presented in **R. Schmidt and P. Pfeifer (1989)(b)**. In this work, a two level process control scheme to determine both process mean and the upper control limit is presented. The paper also compares the cost saving with that of a one level of process control.

In the extension of **T. Boucher and M. Jafari (1991)** the W. Hunter and C. kartha's (1977) work was extended and single sampling inspection is introduced in the model.

D. Golhar and S. Pollock (1992) examined the effect of variance reduction on the cost for the D. Golhar and S. Pollock (1988) model and gave a close form approximate solution.

In **Brain J. Melloy (1991)**, the problem of a uniform filling of an item is formulated under compliance testing. The objective was to minimize the non-compliance and give away cost.

Do sun Bai and Min Koo lee (1993) presented the problem of selecting the process mean and the cut-off value of a correlated variable for a filling process in which inspection is based on the correlated variable rather than the process mean itself.

In **F. J. Arcelus and M. A. Rahim (1994)** a model for simultaneously selecting the optimal target means for both the variable and attribute quality characteristic is presented. Optimality conditions are derived and a computational algorithm is given.

K. S. Al-Sultan (1994) addressed the problem of two machines in series where a sampling plan is used. An algorithm for finding optimal machine parameters for the two machines in series case, with single sampling inspection at each machine is given.

Shaul P. Ladeny (1995) assumed the process in which oversized items and undersized items are repaired at different costs. The objective is to maximize the profit.

D. P. Mihalko and D. Golhar (1995) was the first to consider the process variance as an unknown parameter. In this paper, a method for the determination of the confidence interval for the optimal process setting for the case of an unknown process variance is proposed.

Liu, Tang and Chun (1995) considered the case of a filling process with limited capacity constraint. The optimal process parameters to be determined are process mean and upper specification limit.

F. J. Arcelus (1996) presented two models. The paper introduces the Taguchi quadratic loss function for the targeting problem. In the first model, the target mean is a trade off value between the target mean of the society and the mean that maximizes/minimizes profit/cost for the producer.

In F. J. Arcelus and M. A. Rahim (1996), four models are presented for different assumption related to finding a trade off between conflicting objectives of conformity and uniformity. A quadratic penalty is used for the uniformity of the product.

M. K. Lee and J. S. Jang (1997) introduced the case of the three-class screening. In this paper, it is assumed that the products are sold in two different markets with different price structures. Two models were presented. In the first model, the objective is to find the optimal mean when inspection is based on the same quality characteristic. While in the second model it is assumed that the inspection is based on a correlated variable.

M. F. Pulak and K. S. AL-Sultan (1997). In this paper, a computer program is presented for nine different "Process Targeting" models.

K. S. AL-Sultan and M. A. Al-Fawzan (1997)(a) is an extension of M. A. Rahim and P. K. Banerjee's (1988) model. The paper assumed a process with random linear drift and is assumed to have both upper and lower specification limits. The objective is to find the optimal initial mean and cycle length. Variance of the process is assumed known and constant.

K. S. Al-Sultan and M. F. Pulak (1997) presented a model for finding the optimal mean of a filling process under rectifying inspection. The effect of variance reduction is also considered for the case.

K. S. AL-Sultan and M. A. Al-Fawzan (1997)(b) studied the model of M. A. Rahim and P. K. Banerjee (1988) i.e., systems with linear drift for the case of variance reduction and optimal initial process mean and cycle time is given.

J. Roan, L. Gong, K. Tang (1997) considers other production decisions such as production setup and raw material procurement policies. Two discount rate policies for the inventory are adopted. The production rate is assumed to be the function of the mean of the process.

M. Cain and C. Janssen (1997) presented the model where the cost is asymmetric across the target. A linear cost below lower specification limit and a quadratic cost above specification limit are assumed.

S. Pollock, D. Golhar (1998). In this paper the canning process with constant demand and capacity constraint for the production process is considered. The model also assumes a penalty for producing a non-conforming item.

P. E. Pfeifer (1999) provided a general piecewise linear model for the canning model.

Sung Hoon Hong & E. A. Elsayed (1999) studied the effect of measurement error on the optimal mean settings for the case of two class screening situation.

Figure 2-1 shows the development of the targeting problem over the years. The literature has no model that incorporates Taguchi quadratic loss function and inspection error. This is the focus of the thesis.

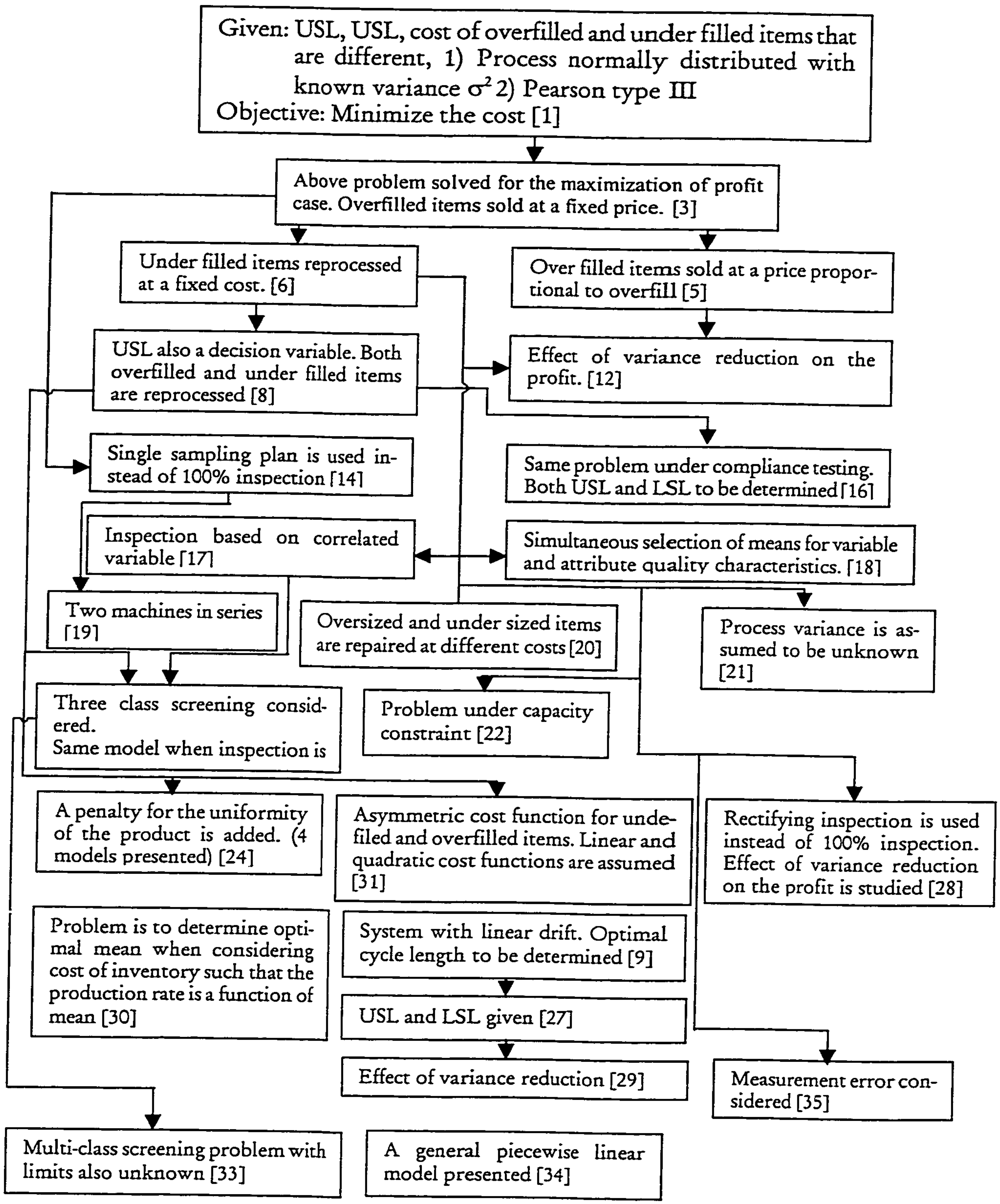


Figure 2-1: "Process Targeting" Problem Development

2.3 MODEL 1 (EPM1)

As this thesis work extends the work done by Min Koo Lee and Joong Soon Jang (1997), their model (the model will be referred to as "Model 1" or "EPM1") is presented in this chapter. This model is used as a basis for the extension made in this thesis.

2.3.1 MODEL DESCRIPTION

The model in their paper applies to the production processes that are filling a can or turning a metal bar etc. The quality characteristics 'Y' could be the net weight of the can or the mean diameter of the turned metal bar. The quality characteristic, in this model, is assumed to be normally distributed. The product, out of the production process, is classified into three classes based on product specifications. Considering can filling example, class one (grade 1) is the product that has a net weight $\geq L_1$. The second-class product (grade 2) is the product that has the net weight between L_2 and L_1. The third class is scrape, which has a net weight $< L_2$.

The objective of the model is to find the optimal location of the process mean, to maximize the profit resulting from grade 1 and grade 2. The model assumptions are as follows

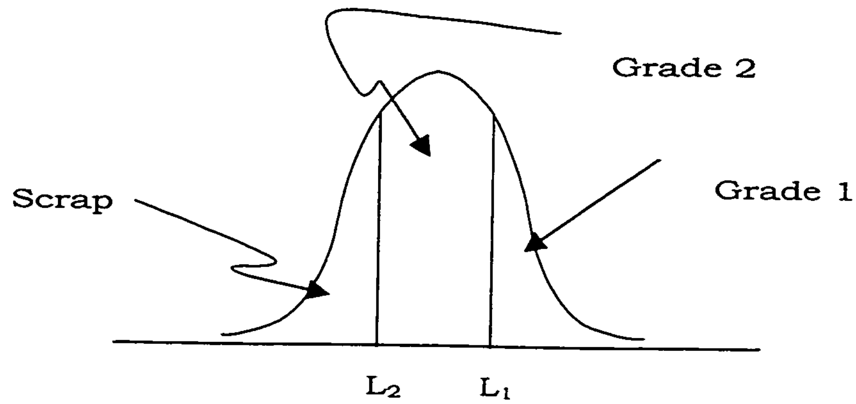


Figure 2-2: A production process with multi class screening

2.3.2 MODEL ASSUMPTIONS

The assumptions of the models are:

1. A single item is to be sold in two different markets with different cost/profit structures.
2. The inspection process is error free.
3. 100% sampling is used.
4. The quality characteristic 'Y' is assumed normally distributed with unknown process mean μ_y and known variance σ_y^2.
5. The inspection is based on 'Y'.
6. $a_1 > a_2 > r$.
7. $L_1 > L_2$
8. The production cost per item is $c_0 + cy$
9. Specification limits on different grade are:

Specification limits on 'Y' for grade 1 are	$Y \geq L_1$,
Specification limits on 'Y' for grade 2 are	$L_2 \leq Y < L_1$,
Specification limits on 'Y' for scrape are	$Y < L_2$,

2.3.3 MODEL FORMULATION

Now the profit (per unit e.g., per kilogram etc.) function $P(y)$ can be expressed as:

$$P(y)=\begin{cases} a_1-(c_0+cy+c_i) & Y \geq L_1 \\ a_2-(c_0+cy+c_i) & L_2 \leq Y < L_1 \\ r-(c_0+cy+c_i) & Y \geq L_2 \end{cases}$$

Now, The per unit expected profit can be written as:

$$E\{P(y)\}=\int_{L_1}^{\infty}[a_1-(c_0+cy+c_i)]f(y)dy+\int_{L_2}^{L_1}[a_2-(c_0+cy+c_i)]f(y)dy \\ +\int_{-\infty}^{L_2}[r-(c_0+cy+c_i)]f(y)dy \qquad (3\text{-}1)$$

where $f(y)$ is the distribution of the quality characteristic 'Y'. Equation (3-1) can be simplified as:

$$\Rightarrow \quad E\{P(y)\}=(a_1-c_0-c_i)\int_{L_1}^{\infty}f(y)dy+(a_2-c_0-c_i)\int_{L_2}^{L_1}f(y)dy \\ +(r-c_0-c_i)\int_{-\infty}^{L_2}f(y)dy-c\int_{L_1}^{\infty}yf(y)dy-c\int_{L_2}^{L_1}yf(y)dy-c\int_{-\infty}^{L_2}yf(y)dy$$

$$\Rightarrow \quad E\{P(y)\}=a_1\int_{L_1}^{\infty}f(y)dy+a_2\int_{L_2}^{L_1}f(y)dy+r\int_{-\infty}^{L_2}f(y)dy-(c_0+c_i)\int_{-\infty}^{\infty}f(y)dy \\ -c\int_{-\infty}^{\infty}yf(y)dy$$

$$\Rightarrow \quad E\{P(y)\}=a_1\int_{L_1}^{\infty}f(y)dy+a_2\int_{L_2}^{L_1}f(y)dy+r\int_{-\infty}^{L_2}f(y)dy-(c_0+c_i)-c\mu \qquad (3\text{-}2)$$

Using the normality assumption i.e.,

$$f(y) \sim N(y;\mu,\sigma_y^2)$$

and letting:

$$z = \frac{y-\mu}{\sigma_Y}$$

$$\Rightarrow \qquad y = \mu + z\sigma_Y$$

For standard normal the limits can be redefined as

at $y = L_1$ $\qquad z = \frac{L_1 - \mu}{\sigma_Y} = \Gamma_1$

at $y = L_2$ $\qquad z = \frac{L_2 - \mu}{\sigma_Y} = \Gamma_2$

Now, assuming $\phi(y)$, $\Phi(y)$ and $\phi(z)$, $\Phi(z)$ be the p.d.f. and c.d.f. of the normal and standard normal distributions i.e.,

$$\varphi(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma_Y}\right)^2}, \qquad \varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

$$\int_{y_1}^{y_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma_Y}\right)^2} dy = \Phi(y_2) - \Phi(y_1), \qquad = \int_{z_1}^{z_2} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz = \Phi(z_2) - \Phi(z_1)$$

Using the relationship '$\mu = L_l - \Gamma_l\sigma_y$' and the relationships for standard normal distribution, also, letting $E\{P(y)\} \equiv EPM1$ we can rewrite the equation (3-2) as:

$$EPM1 = a_1[\Phi(-\Gamma_1)] + a_2\Phi(\Gamma_1) + (r - a_2)\Phi(\Gamma_2) - c_0 - c_i - c(L_l - \Gamma_l\sigma_Y) \quad (3\text{-}3)$$

This is the form presented in M. K. Lee and J. S. Jang (1997). The model is used to determine the process mean.

2.4 CONCLUSION

In this chapter, the literature in the area of process targeting is reviewed. Then the model of Min Koo Lee and Joong Soon Jang (1997) is presented. The work in this thesis extends the model of Min Koo Lee and Joog Soon Jang (1997) in two directions. First, the model is extended by incorporating errors in the measurement systems and second the concept of product consistency is incorporated through Taguchi loss function.

The next chapter extends the model by incorporating measurement errors. In case of measurement error, the observed quality characteristic is not the true value and this may regime modifying inspection criteria to counter the effect of the error. This issue is addressed in the next chapter in detail.

CHAPTER 3

PROCESS TARGETING WITH MULTI-CLASS SCREENING AND MEASUREMENT ERROR

3.1 INTRODUCTION

The purpose of this chapter is to extend model 1, presented in chapter 2, to the case where the measurement system is considered error prone. The motivation behind this extension stems from the fact that measurement system can cause considerable loss due to the misclassification of the product. The loss could be of replacement and warranty costs, loss of good will, and loss of profit by selling a higher-grade product as lower grade due to misclassification etc. The per unit loss in profit due to this error may seem small, however in some cases, the over all loss of profit (consider a million units produced per year) may be in millions.

In this chapter, we assume that the quality characteristic is measured and the observed value of the quality characteristic 'X' is not the same as the true value 'Y' due to the presence of measurement error, i.e.,

$$X = Y + \varepsilon$$

Where 'ε' represents error in measurement and it is assumed to be unbiased and normally distributed with known variance σ_ε^2 i.e., $\varepsilon \sim N(0, \sigma_\varepsilon^2)$.

For reducing the effect of the error, "cut off points" for the measurement system are considered i.e., instead of the product, being inspected based on the limits of the grades, these cut off points will be used as the criteria of classification, see figure 3-1. Here the cut off points are considered as decision variables. The location of these cut off points depends on many factors e.g., 1) the loss in profit for misclassifying a higher grade product into a lower grade, & 2) the penalty associated with misclassification of a lower grade product classified as a higher grade product 3) position of the mean etc. If the loss due to misclassifying e.g., grade 2 items as grade 1, is higher than loss of profit if grade 1 is misclassified as grade 2, than the cut off point may set higher than L_1 in order to reduce the chances of misclassifying the grade 2 product in grade 1. However, its value is also dependent on the process mean (how close the mean is to the specification limits).

The objective of this model is to maximize the expected profit by finding the optimal process mean and reducing the effect of error, on the expected profit, by finding the optimal cut off points. The rest of the chapter is organized as follows: in the section 3.2, the model development will be discussed, followed by the illustrative

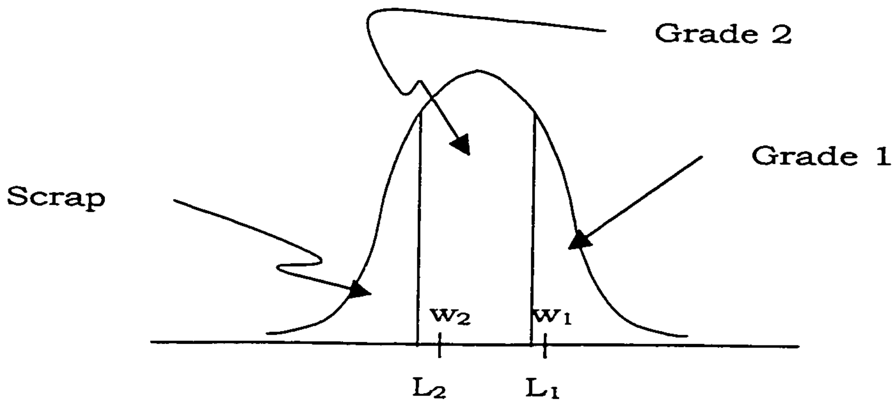


Figure 3-1: A production process with multi class screening. w_i show the cut off values for the inspection.

example and sensitivity analysis in section 3.3. The conclusion will be presented in section 3.4.

3.2 MODEL DEVELOPMENT

In this section, the new model developed for multi-class screening targeting model under measurement system with errors will be presented. At First, the model assumptions will be presented, next, the relationship between the observed values and the actual quality characteristic will be discussed, and finally the model will be presented.

3.2.1 MODEL ASSUMPTION

The assumptions of the models are:

1. A single item is to be sold in two different markets with different cost/profit structures.
2. The inspection process is error prone.
3. 100% sampling is considered.
4. The measurement 'X' is assumed to be unbiased and distributed normally across the true value.
5. The measuring instrument is assumed to have a known variance.
6. The quality characteristic 'Y' is assumed normally distributed with unknown process mean μ_y and known variance σ_y^2.
7. The inspection is based on 'X'.
8. $a_1 > a_2 > r$.
9. $L_1 > L_2$
10. The production cost per item is $c_0 + cy$

Specification limits on different grade are:

Specification limits on 'Y' for grade 1 are $Y \geq L_1$,

Specification limits on 'Y' for grade 2 are $L_2 \leq Y < L_1$,

Specification limits on 'Y' for scrap are $Y < L_2$,

3.2.2 RELATIONSHIP BETWEEN 'Y' AND 'X'

Here the 'X' is the observed value of 'Y'

i.e.,

$$X = Y + \varepsilon$$

Where 'ε' is error in measurement and it is assumed to be unbiased and normally distributed i.e., $\varepsilon \sim N(0, \sigma_\varepsilon^2)$

The expected value of the observed value 'X' will be

$$E(X) = E(Y + \varepsilon)$$

$$E(X) = E(Y) + E(\varepsilon)$$

$$E(Y) = \mu$$

Similarly, the variance of 'X' can derived as

$$\mathrm{Var}(X) = \mathrm{Var}(Y + \varepsilon)$$

$$\mathrm{Var}(X) = \mathrm{Var}(Y) + \mathrm{Var}(\varepsilon) + 2\mathrm{Cov}(Y, \varepsilon)$$

Since Y and ε are independent $\mathrm{Cov}(y, \varepsilon) = 0$. Rewriting the above equation:

$$\mathrm{Var}(X) = \mathrm{Var}(Y) + \mathrm{Var}(\varepsilon)$$

$$\sigma_X^2 = \sigma_Y^2 + \sigma_\varepsilon^2$$

Assuming 'X' and 'Y' are normally distributed, the joint distribution of 'X' and 'Y' can be written as:

$$h(x, y) = \psi(x, y) = g(x / y; y + \varepsilon, \sigma_X^2) f(y; \mu, \sigma_Y^2)$$

$$\psi(y, x) = \frac{1}{2\pi\sigma_y\sigma_\varepsilon\sqrt{1-\rho^2}} e^{-\frac{1}{2\sqrt{1-\rho^2}}\left\{\left(\frac{x-\mu}{\sigma_X}\right)^2 + \left(\frac{y-\mu}{\sigma_Y}\right)^2 - 2\rho\frac{x-\mu}{\sigma_X}\frac{y-\mu}{\sigma_Y}\right\}}$$

In standard form:

$$\psi(v, u) = \frac{1}{2\pi} e^{-\frac{1}{2}\left\{(u)^2 + (v)^2 - 2\rho uv\right\}}$$

Let ρ = correlation co-efficient. Where

$$\rho = 1 - \frac{\sigma_\varepsilon^2}{\sigma_X^2} = \frac{\sigma_Y^2}{\sigma_X^2} = \frac{\sigma_\varepsilon^2}{\sigma_Y^2 + \sigma_\varepsilon^2}$$

3.2.3 STATEMENT OF THE MODEL

Assuming w_i be the cut off value on 'X' therefore if:

$X \geq w_1$	we conclude	$Y \geq L_1$
$w_2 \leq X < w_1$	we conclude	$L_2 \leq Y < L_1$
$X < w_2$	we conclude	$Y < L_2$

The penalties associated in misclassification due to measurement error are:

b_{21}	=	Penalty such that	$X \geq w_1$	while $L_2 \leq Y < L_1$
b_{s1}	=	Penalty such that	$X \geq w_1$	while $Y < L_2$
b_{s2}	=	Penalty such that	$w_2 \leq X < w_1$	while $Y < L_2$

Now the profit function can be written as:

$$P(y) = \begin{cases} a_1 - (c_0 + cy + c_i) & X \geq w_1 & Y \geq L_1 \\ a_1 - (c_0 + cy + c_i) - b_{21} & X \geq w_1 & L_2 \leq Y < L_1 \\ a_1 - (c_0 + cy + c_i) - b_{s1} & X \geq w_1 & Y < L_2 \\ a_2 - (c_0 + cy + c_i) & w_2 \leq X \leq w_1 & L_2 \leq Y < L_1 \\ a_2 - (c_0 + cy + c_i) & w_2 \leq X \leq w_1 & Y \geq L_1 \\ a_2 - (c_0 + cy + c_i) - b_{s2} & w_2 \leq X \leq w_1 & Y < L_2 \\ r - (c_0 + cy + c_i) & X \leq w_2 & Y < L_2 \\ r - (c_0 + cy + c_i) & X \leq w_2 & Y \geq L_1 \\ r - (c_0 + cy + c_i) & X \leq w_2 & L_2 \leq Y < L_1 \end{cases}$$

The profit function shows three possibilities for each grade (see Table 3.1) i.e., for grade 1: the possibilities are 1) to be classified correctly, 2) grade 1 misclassified as grade 2 & 3) grade 1 misclassified as scrap. Similarly for grade 2: the possibilities are 1) to be classified correctly, 2) grade 2 misclassified as grade 1 & 3) grade 2 misclassified as scrap. For scrap: the possibilities are 1) to be classified correctly, 2) scrap misclassified as grade 1 & 3) scrap misclassified as grade 2. If a lower grade item is misclassified as higher grade, the loss is represented by the penalties associated with it i.e., b_{ij}, on the other hand if a higher grade item is misclassified as a lower grade the loss is reflected in the loss of profit i.e., selling at lower price.

Therefore, the expected profit per item can be written as:

$$
\begin{aligned}
E\{P(\mu, w_1, w_2)\} = & \int_{w_1}^{\infty}\int_{L_1}^{\infty}(a_1 - c_0 - cy - c_i)\varphi(x,y)dydx \\
& + \int_{w_1}^{\infty}\int_{L_2}^{L_1}(a_1 - c_0 - cy - c_i - b_{21})\psi(x,y)dydx + \int_{w_1}^{\infty}\int_{-\infty}^{L_2}(a_1 - c_0 - cy - c_i - b_{s1})\psi(x,y)dydx \\
& + \int_{w_2}^{w_1}\int_{L_2}^{L_1}(a_2 - c_0 - cy - c_i)\psi(x,y)dydx + \int_{w_2}^{w_1}\int_{L_1}^{\infty}(a_2 - c_0 - cy - c_i)\psi(x,y)dydx \\
& + \int_{w_2}^{w_1}\int_{-\infty}^{L_2}(a_2 - c_0 - cy - c_i - b_{s2})\psi(x,y)dydx + \int_{-\infty}^{w_2}\int_{-\infty}^{\infty}(r - c_0 - cy - c_i)\psi(x,y)dydx
\end{aligned}
$$

Simplifying the above expression,

$$
\begin{aligned}
E\{P(\mu, w_1, w_2)\} = a_1 & \int_{w_1}^{\infty}\int_{-\infty}^{\infty}\psi(x,y)dydx + a_2\int_{w_2}^{w_1}\int_{-\infty}^{\infty}\psi(x,y)dydx \\
& + r\int_{-\infty}^{w_2}\int_{-\infty}^{\infty}\psi(x,y)dydx - b_{21}\int_{w_1}^{\infty}\int_{L_2}^{L_1}\psi(x,y)dydx \\
& - b_{s1}\int_{w_1}^{\infty}\int_{-\infty}^{L_2}\psi(x,y)dydx - b_{s2}\int_{w_2}^{w_1}\int_{-\infty}^{L_2}\psi(x,y)dydx \\
& - (c_0 + c_i)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\psi(x,y)dydx - c\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\psi(x,y)dydx
\end{aligned}
$$

$$
\begin{aligned}
E\{P(\mu, w_1, w_2)\} = a_1 & \int_{w_1}^{\infty}\varphi(x)dx + a_2\int_{w_2}^{w_1}\varphi(x)dx + r\int_{-\infty}^{w_2}\varphi(x)dx \\
\Rightarrow \quad & - b_{21}\int_{w_1}^{\infty}\int_{L_2}^{L_1}\psi(x,y)dydx - b_{s1}\int_{w_1}^{\infty}\int_{-\infty}^{L_2}\psi(x,y)dydx \qquad (3\text{-}1) \\
& - b_{s2}\int_{w_2}^{w_1}\int_{-\infty}^{L_2}\psi(x,y)dydx - c_0 - c_i - c\mu
\end{aligned}
$$

In order to use standard normal and bivariate normal distributions in equation (3-1), redefining the limits.

Let:

$$u = \frac{x-\mu}{\sigma_x}, \qquad v = \frac{y-\mu}{\sigma_y}$$

at $y = L_1$ $\qquad v = \dfrac{L_1 - \mu}{\sigma_y} = \Gamma_1$

at $y = L_2$ $\qquad v = \dfrac{L_2 - \mu}{\sigma_y} = \Gamma_2$

at $x = w_1$ $\qquad u = \dfrac{w_1 - \mu}{\sqrt{\sigma_y^2 + \sigma_e^2}} = \delta_1$

at $x = w_2$ $\qquad u = \dfrac{w_2 - \mu}{\sqrt{\sigma_y^2 + \sigma_e^2}} = \delta_2$

Using the relationship '$\mu = L_1 - \eta\sigma_y$' and the relationships for standard normal and bivariate normal distributions, also, letting $E\{P(\mu, w_1, w_2)\} \equiv EPM2$ we can rewrite the equation (3-1) as:

$$\begin{aligned} EPM2 = {} & a_1\Phi(-\delta_1) + a_2[\Phi(\delta_1) - \Phi(\delta_2)] + r\Phi(\delta_2) \\ & - b_{21}\int_{\delta_1}^{\infty}\int_{\Gamma_2}^{\Gamma_1}\psi(v,u)dvdu - b_{s1}\int_{\delta_1}^{\infty}\int_{-\infty}^{\Gamma_2}\psi(v,u)dvdu - b_{s2}\int_{\delta_2}^{\delta_1}\int_{-\infty}^{\Gamma_2}\psi(v,u)dvdu \\ & - c_0 - c_i - c(L_1 - \Gamma_1\sigma_y) \end{aligned} \qquad (3\text{-}2)$$

Where $\int_{-\infty}^{z}\varphi(x)dx$ is represented as $\Phi(z)$ in standard form. This is the expression for the expected value of profit for a multi-class screening quality system under error prone measurement system.

3.3 RESULTS AND ANALYSIS

In this section, an illustrative example will be presented. This will be followed by the sensitivity analysis. For the numerical analysis, 'FindMinimum' function of Mathematica 4.0 is used. The function uses modified Powell's method. The analysis is performed on a Compaq Deskpro, Pentium III computer with 64 MB of RAM. See appendix B for the notebook (program code).

3.3.1 ILLUSTRATIVE EXAMPLE

Consider a packing plant of a cement factory. The plant consists of two processes a filling process and an inspection process. Each cement bag processed by filling machine is moved to the loading and dispatching phase on the conveyor belt. Inspection is performed by automatic weighing system. Suppose that the cost components and the specification limits are a_1=\$5.5, a_2=\$5.1, r=\$2.5, c_0=\$0.1, c_i=\$0.04, c=\$0.01, b_{s1}=\$5, b_{s2}=\$5.5, b_{21}=\$5, L_1=41.5 kg, L_2=40.0 kg and $\sigma_y^2 = (1.25)^2$. The error in the measuring system is represented by the correlation co-efficient having the value $\rho = 0.85$ i.e., $\sigma_\varepsilon^2 = (0.525)^2$.

The result obtained by model 1 (no error assumed) is as follows:

E(p) = \$ 5.178

μ = 44.846 kg

The expected profit and the optimal values of the mean and the cut off values by model 2 are found out to be:

E(p) = \$ 5.176,

μ = 45.6795

w_1 = 41.1746

w_2 = 37.7737

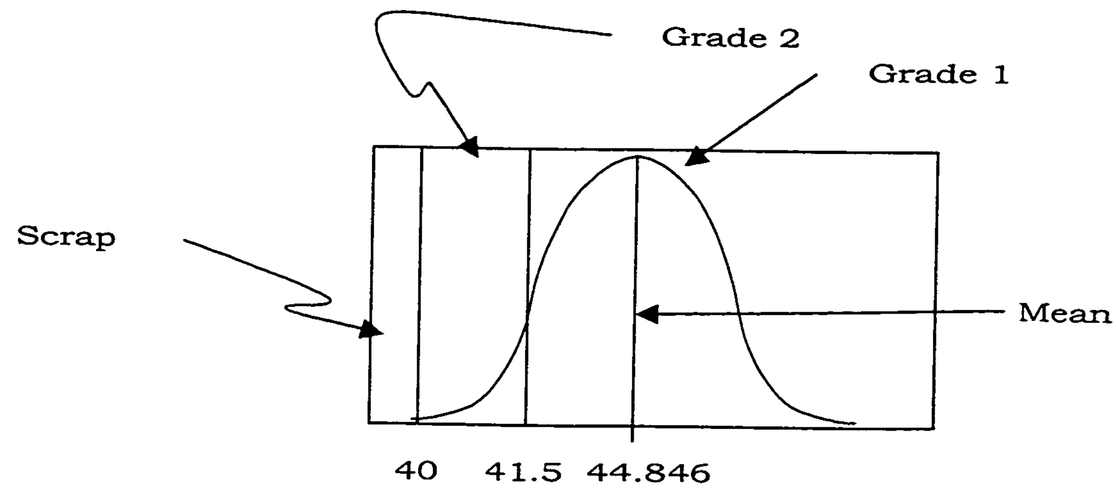


Figure 3-2 (a): Optimal mean settings obtained from model 1

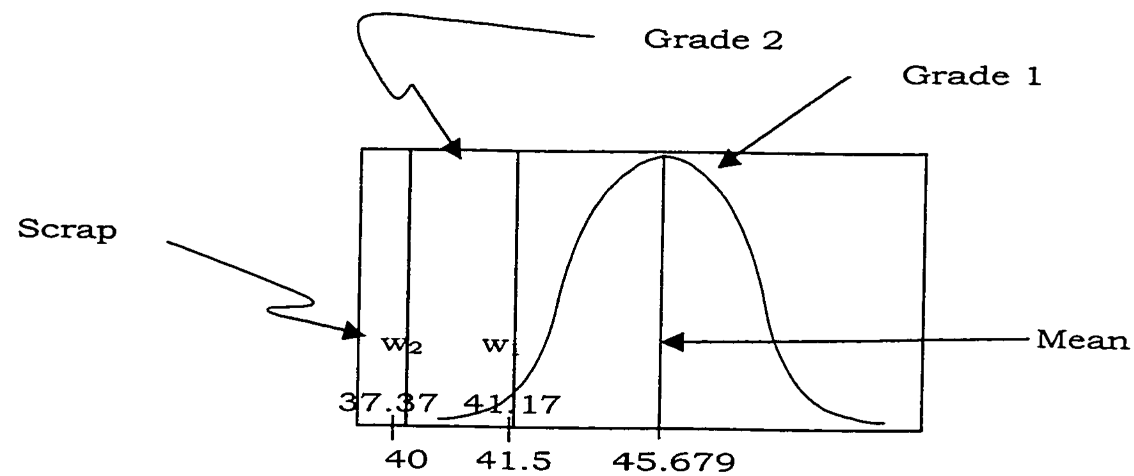


Figure 3-2 (b): Optimal mean and cut off point settings obtained from model 2

The expected profit is almost same as the one given by model 1 i.e., the model 2 quite successfully nullified the effect of measurement error by finding the optimal values for mean and cut off points.

The mean, in model 2 result, is set higher than L_1, as well as the mean obtained by model 1, showing that the process will be forced to produce more of grade 1 products, also, reducing the chances of product falling near specification limits (lesser misclassification). In addition, as w_1 is set below L_1, the model shows that it is better to misclassify more grade 2 products as grade 1 and to pay the penalty than lose the profit by misclassifying grade 1 as grade 2. The situation is the same for the scrap to grade 2 case i.e., the result shows that it is better to pay penalty for misclassifying than losing profit, however the effect of this part is significantly less as compared to the first because mean is set considerably higher than the specification limit L_2. As the level of w_1 is also dependent on mean the location of w_1 may fall at both sides of the L_1.

Now, in order to determine per unit loss in profit, the mean obtained by model 1 is substituted in model 2 and w_1 & w_2 are taken at L_1 & L_2. The per unit profit is found out to be:

E(p) = $ 4.89946

As can be seen clearly, If the model 1 is used instead of model 2, although the apparent expected profit was $ 5.178, in reality the expected profit will be only $ 4.89946 i.e., there is net expected gain of $ 0.27854 if model 2 is used. Now consider a plant producing million units per year, the net expected gain in profit per year will be $ 278540 if settings from model 2 will be used.

3.3.2 SENSITIVITY ANALYSIS

In this sensitivity analysis, the effect of different parameters on the output values i.e., the expected profit, the optimal mean & the optimal cut off values are studied. As there are a number of parameters, a few of them were chosen. These parameters are: $\frac{c}{a_1-a_2}, \frac{a_2-r}{a_1-a_2}, \sigma_y$ and ρ. These are the parameters used in Min Koo Lee & Joon Soong Jang (1997). Here $\frac{c}{a_1-a_2}$ represents the 'per unit production cost' in dimensionless from, $\frac{a_2-r}{a_1-a_2}$ represents per item selling prices in the same manner, ρ is correlation coefficient that represents the error in measurement system provided no bias in measurement. The main emphasis is on the effect of error i.e., five levels of ρ are taken at 0.85, 0.89, 0.93, 0.97 & 0.995 to cover the effect from low accuracy to high accuracy on the model outputs. Other parameters are taken at two or three levels.

3.3.2.1 EFFECT OF PARAMETERS ON EXPECTED PROFIT

In Table 3-1, the expected profit at different parameter levels is shown. The results show that at a given level of $\frac{c}{a_1-a_2}, \frac{a_2-r}{a_1-a_2}$ & σ_y the effect of the measurement error is minimized, as per unit expected profit remains almost same at different levels of measurement error. This nullifying effect decrease relatively with the increase in per unit production cost. The results can be shows graphically in figures 3-3 to 3-8.

TABLE 3-1: Expected profit at different parameter settings

σ_Y	$\frac{a_2 - r}{a_1 - a_2}$	ρ	$\frac{c}{a_1 - a_2}$ = 0.01	0.15	0.3
1.25	4	0.85	5.08434	1.33681	-2.60536
		0.89	5.08466	1.34457	-2.58819
		0.93	5.08516	1.35517	-2.56597
		0.97	5.08609	1.37154	-2.53327
		0.995	5.08744	1.39233	-2.49326
	6	0.85	5.17579	2.67059	0.03422
		0.89	5.17604	2.67651	0.04683
		0.93	5.17645	2.68442	0.06278
		0.97	5.17716	2.69631	0.08566
		0.995	5.17817	2.71095	0.11302
	8	0.85	5.20786	3.13852	0.96043
		0.89	5.20809	3.14368	0.97124
		0.93	5.20845	3.15050	0.98477
		0.97	5.20907	3.16062	1.00396
		0.995	5.20994	3.17290	1.02664
1.75	4	0.85	5.07466	1.23931	-2.76856
		0.89	5.07509	1.24897	-2.74833
		0.93	5.07578	1.26206	-2.72188
		0.97	5.07702	1.28219	-2.68234
		0.995	5.07879	1.30779	-2.63315
	6	0.85	5.16914	2.60188	-0.08392
		0.89	5.16950	2.60900	-0.06978
		0.93	5.17004	2.61833	-0.05180
		0.97	5.17097	2.63219	-0.02559
		0.995	5.17224	2.64929	0.00649
	8	0.85	5.20231	3.08043	0.85947
		0.89	5.20262	3.08654	0.87135
		0.93	5.20310	3.09443	0.88626
		0.97	5.20390	3.10595	0.90774
		0.995	5.20498	3.11996	0.93380

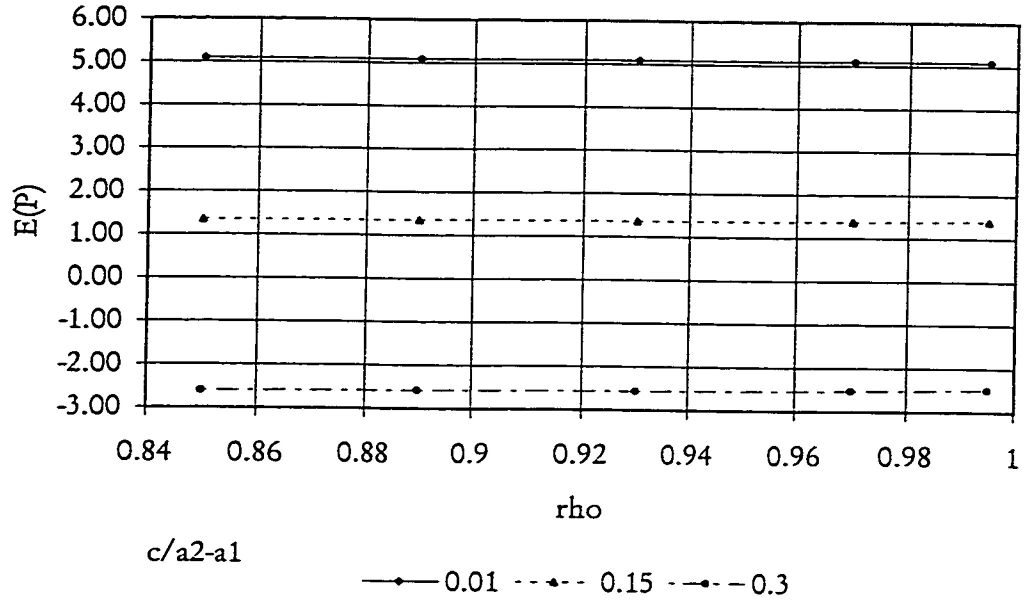


Figure 3-3: E(p) versus (rho) at a2-r/a1-a2 = 4
sigma = 1.25

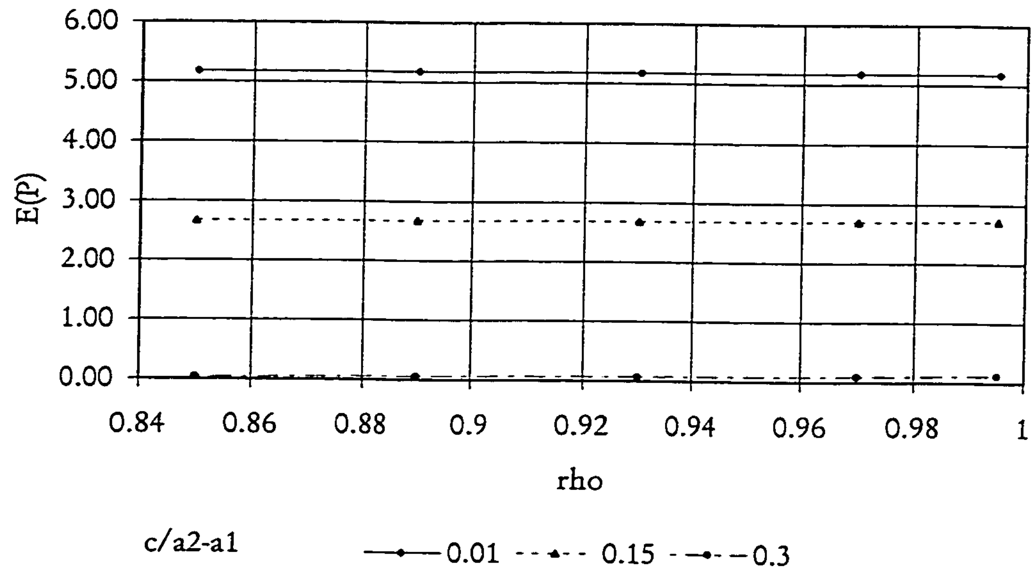


Figure 3-4: E(p) versus (rho) at a2-r/a1-a2 = 6.5
sigma = 1.25

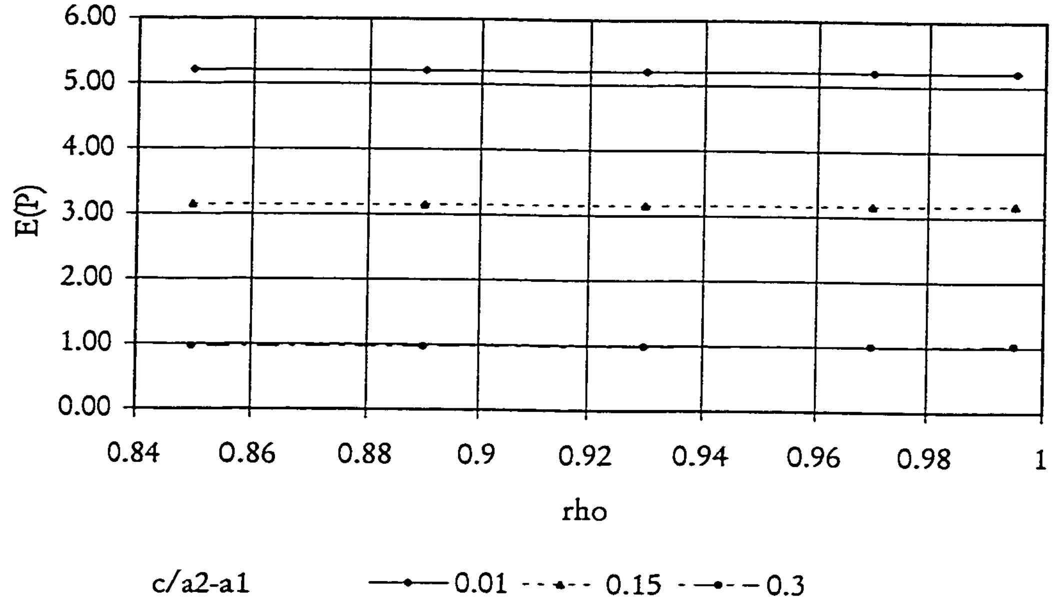


Figure 3-5: E(p) versus (rho) at a2-r/a1-a2 = 8
sigma = 1.25

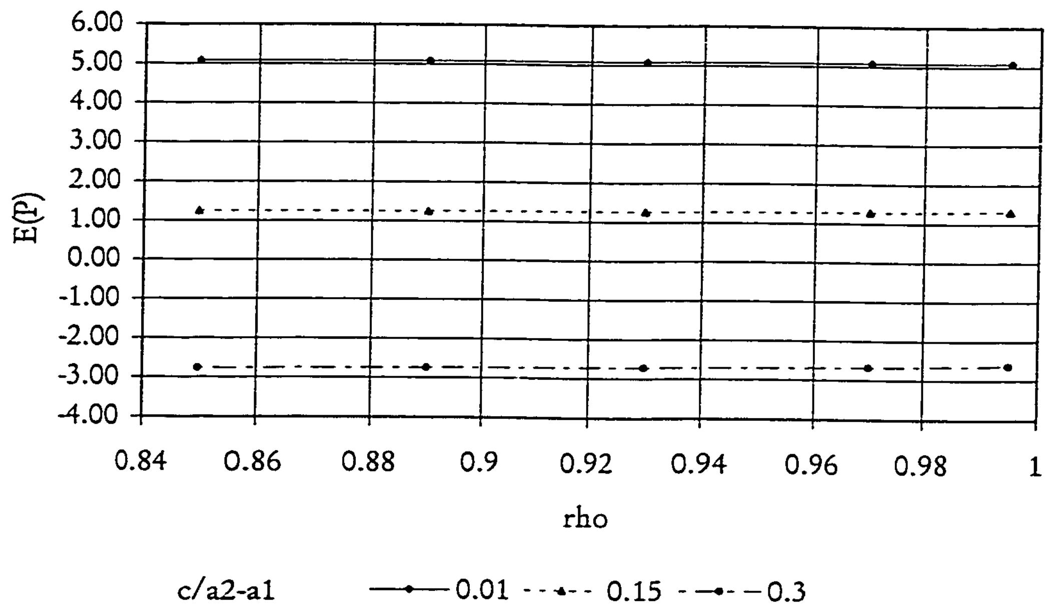


Figure 3-6: E(p) versus (rho) at a2-r/a1-a2 = 4
sigma = 1.75

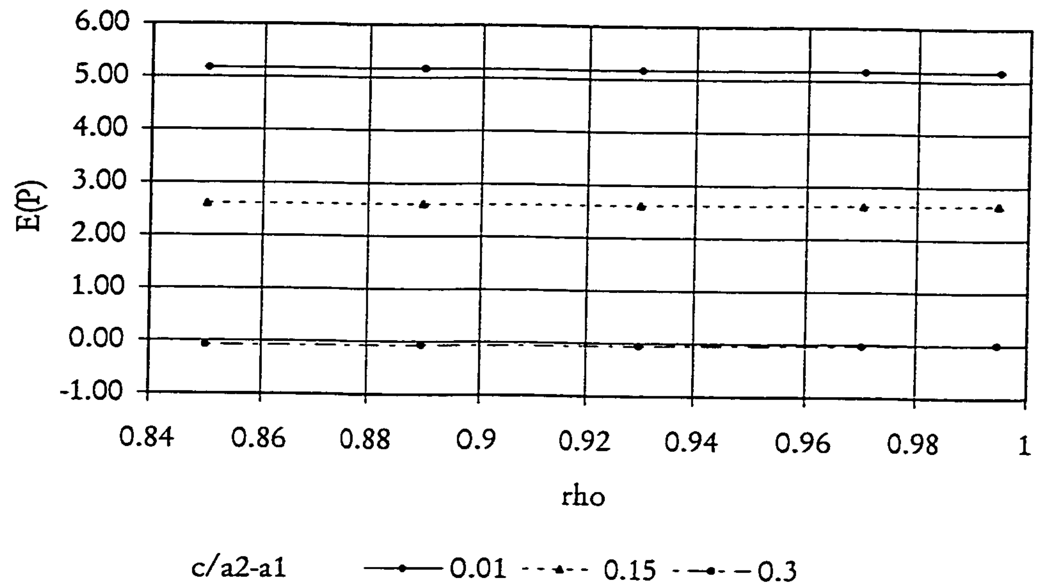


Figure 3-7: E(p) versus (rho) at a2-r/a1-a2 = 6.5
sigma = 1.75

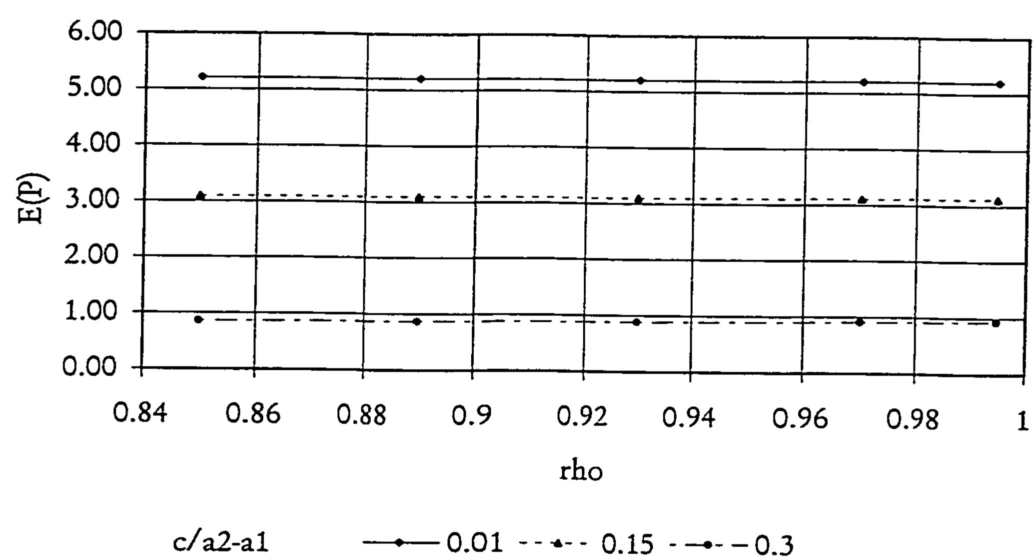


Figure 3-8: E(p) versus measurment error (rho) at a2-r/a1-a2 = 8
sigma = 1.75

Each figure shows per unit expected profit versus measurement error (represented by ρ) containing three plots at three different levels of cost represented by $\frac{c}{a_1 - a_2}$. Per unit, expected profit decreases sharply with cost as can be seen from the figures 3-3 to 3-8. In each figure the level of $\frac{a_2 - r}{a_1 - a_2}$ and σ_y is fixed. The variation in per unit expected profit with $\frac{a_2 - r}{a_1 - a_2}$ at given level of σ_y (two level), as can be seen by comparing figures 3-3 to 3-5 and figures 3-6 to 3-8, is less as compared to $\frac{c}{a_1 - a_2}$.

The results can be summarized as follows:

1. The model performed very well in nullifying the effect of error in measurement (at any given level of $\frac{c}{a_1 - a_2}, \frac{a_2 - r}{a_1 - a_2}$ & σ_y).
2. $\frac{c}{a_1 - a_2}$, significantly reduces per unit expected profit as expected (cost of production is proportional to the level of the process mean).
3. The effect of increase in $\frac{a_2 - r}{a_1 - a_1}$ is less on per expected profit, as compared to $\frac{c}{a_1 - a_2}$. This is because the cost is directly related to mean while the selling prices are proportional to the probability of the product falling in the respective grade.

3.3.2.2 EFFECT OF PARAMETERS ON OPTIMAL MEAN

In Table 3-2, the optimal means obtained, for the same set of problems are presented. The results show that at a given level of $\frac{c}{a_1 - a_2}, \frac{a_2 - r}{a_1 - a_2}$ & σ_y, the mean tends to decrease with increasing ρ. This decrease becomes sharper as $\rho \rightarrow 1$. This shows that, the process mean is forced higher at higher ρ, as the probability of misclassification is higher. These results are shown graphically in figures 3-9 to 3-14. Each figure shows process mean versus measurement error (represented by ρ) containing three plots at three different levels of cost represented by $\frac{c}{a_1 - a_2}$. Process mean decreases sharply with cost as can be seen from the figures 3-9 to 3-14. In each figure the level of $\frac{a_2 - r}{a_1 - a_2}$ and σ_y is fixed. There is an increase in mean with $\frac{a_2 - r}{a_1 - a_1}$ (by comparing figures 3-9 to 3-11 & 3-12 to 3-14 at two different levels of σ_y) .

The results can be summarized as follows:

1. The optimal mean decreases sharply with increase in cost. As the term $-c\mu$ in the model shows, the per unit cost is proportional to the product of production cost and the mean of the process. Consequently, the level of mean goes down with the increase in production cost.

TABLE 3-2: Optimal Mean at different parameter settings

σ_V	$\frac{a_2 - r}{a_1 - a_2}$	ρ	$\frac{c}{a_1 - a_2}$ = 0.01	0.15	0.3
1.25	4	0.85	45.56612	44.11671	43.52502
		0.89	45.50426	44.01552	43.42019
		0.93	45.40914	43.88604	43.29417
		0.97	45.24422	43.69898	43.12043
		0.995	45.01280	43.47492	42.91723
	6	0.85	45.67952	44.24545	43.67366
		0.89	45.60490	44.13367	43.56597
		0.93	45.49309	43.99434	43.43985
		0.97	45.30447	43.79918	43.26914
		0.995	45.04780	43.57193	43.07106
	8	0.85	45.73098	44.30362	43.74098
		0.89	45.65048	44.18742	43.63233
		0.93	45.53110	44.04430	43.50659
		0.97	45.33201	43.84678	43.33766
		0.995	45.06509	43.61990	43.14253
1.75	4	0.85	47.01730	44.95963	44.16257
		0.89	46.93340	44.84148	44.04768
		0.93	46.80595	44.68997	43.90311
		0.97	46.58944	44.46804	43.69367
		0.995	46.29210	44.19644	43.43997
	6	0.85	47.17961	45.16523	44.42265
		0.89	47.07913	45.04206	44.31297
		0.93	46.93210	44.88930	44.17732
		0.97	46.69247	44.67171	43.98156
		0.995	46.37794	44.40942	43.74237
	8	0.85	47.25328	45.25861	44.53898
		0.89	47.14533	45.13376	44.43138
		0.93	46.98979	44.98117	44.29936
		0.97	46.74090	44.76660	44.10943
		0.995	46.42150	44.50979	43.87680

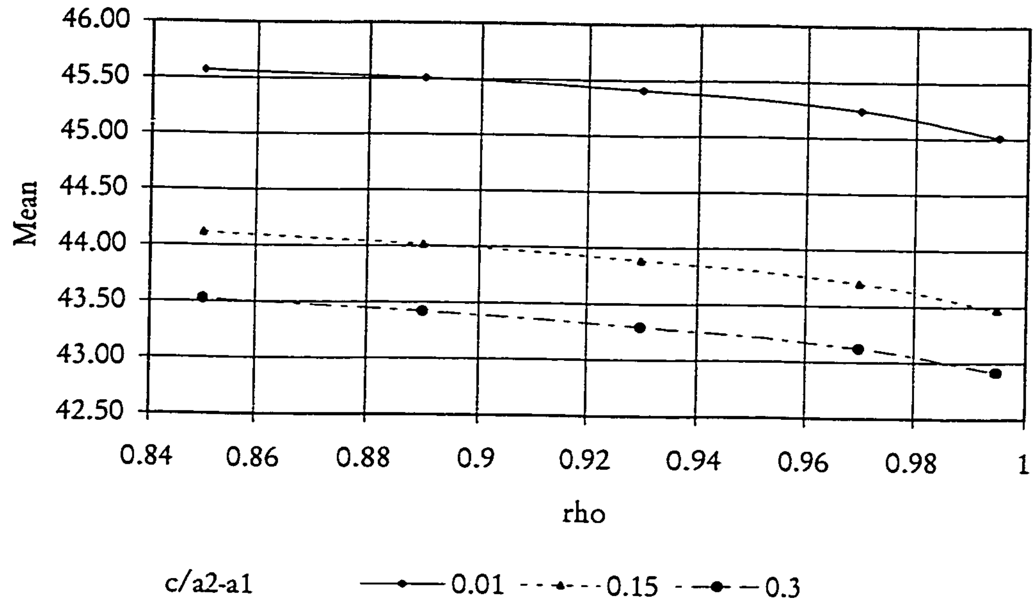


Figure 3-9: Mu versus (rho) at a2-r/a1-a2 = 4
sigma = 1.25

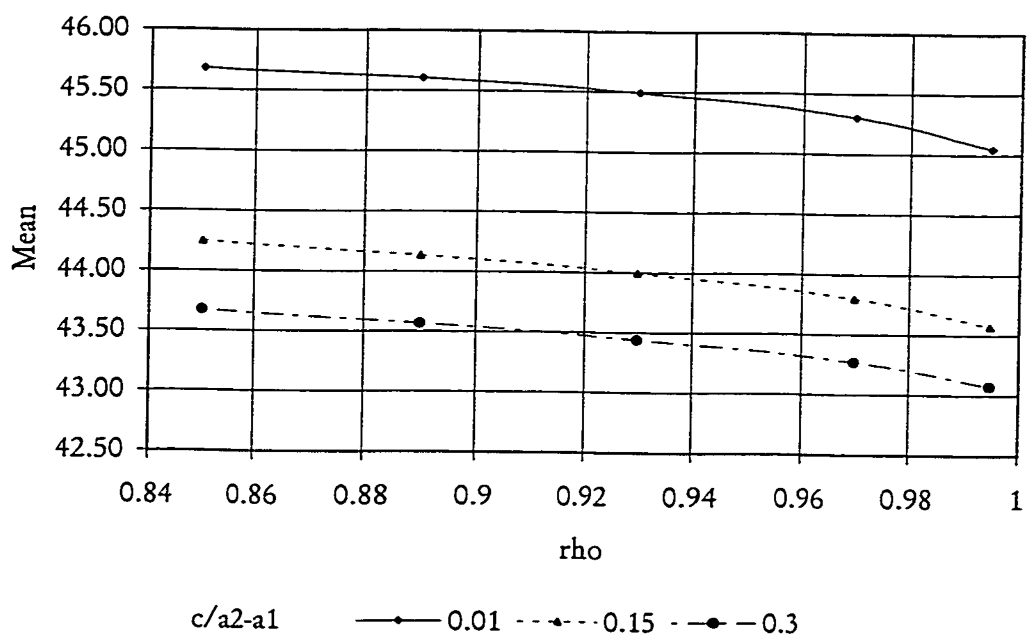


Figure 3-10: Mu versus (rho) at a2-r/a1-a2 = 6.5
sigma = 1.25

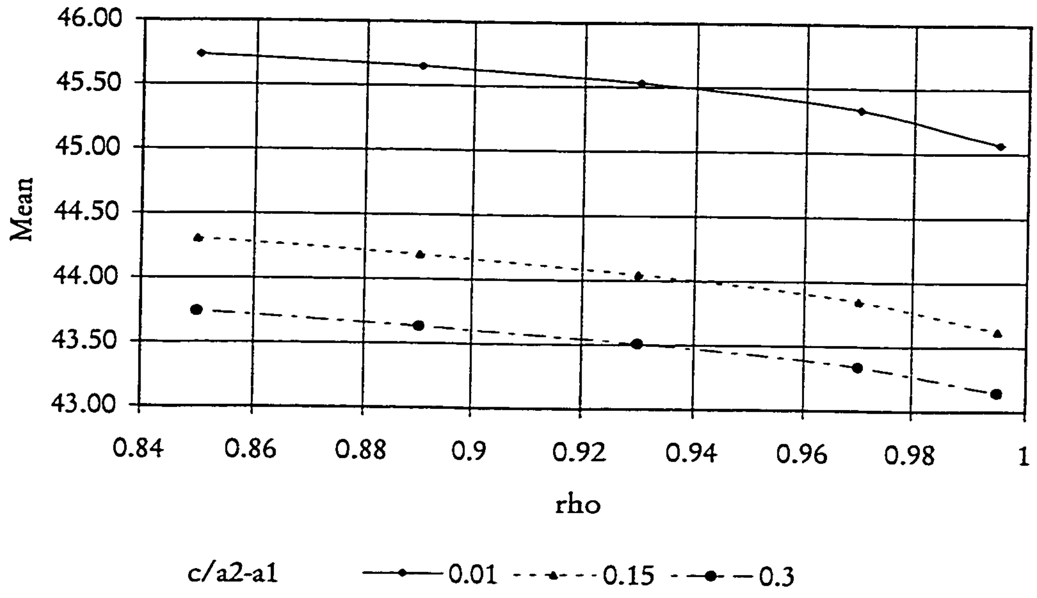


Figure 3-11: Mu versus (rho) at a2-r/a1-a2 = 8
sigma = 1.25

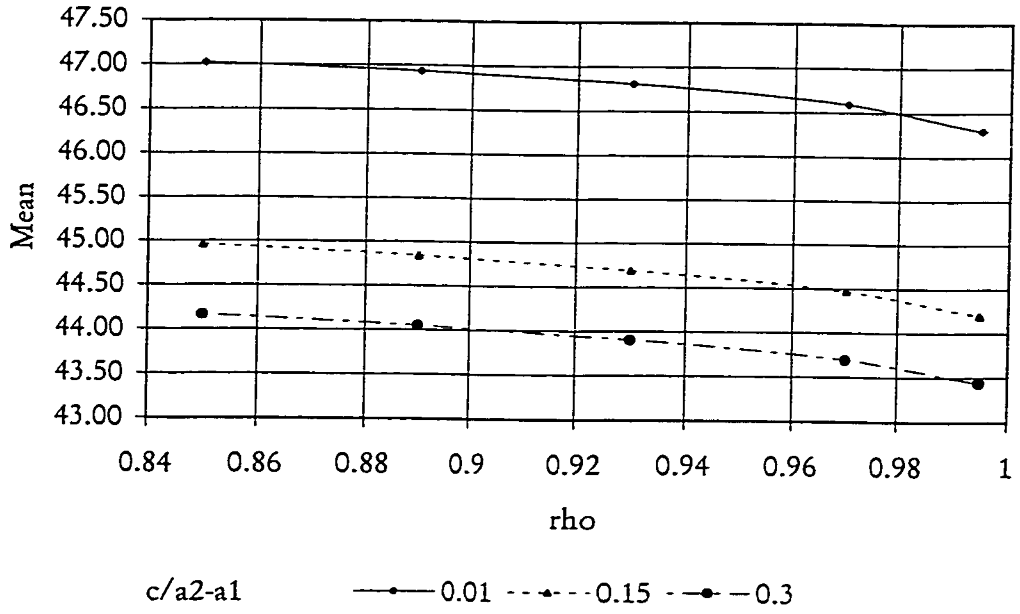


Figure 3-12: Mu versus (rho) at a2-r/a1-a2 = 4
sigma = 1.75

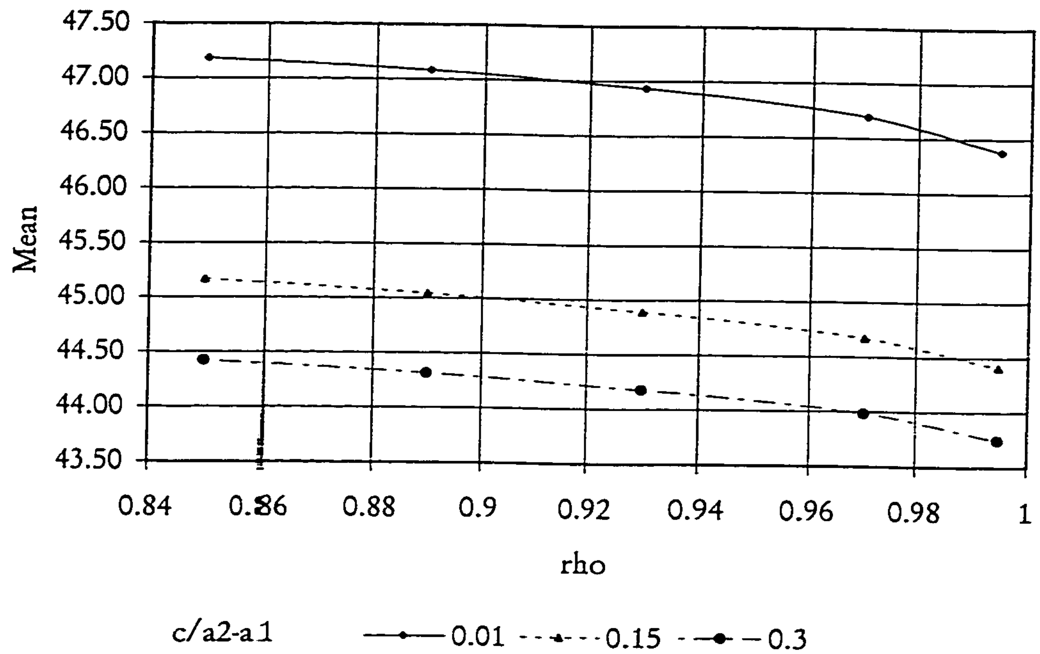


Figure 3-13: Mu versus (rho) at a2-r/a1-a2 = 6.5
sigma = 1.75

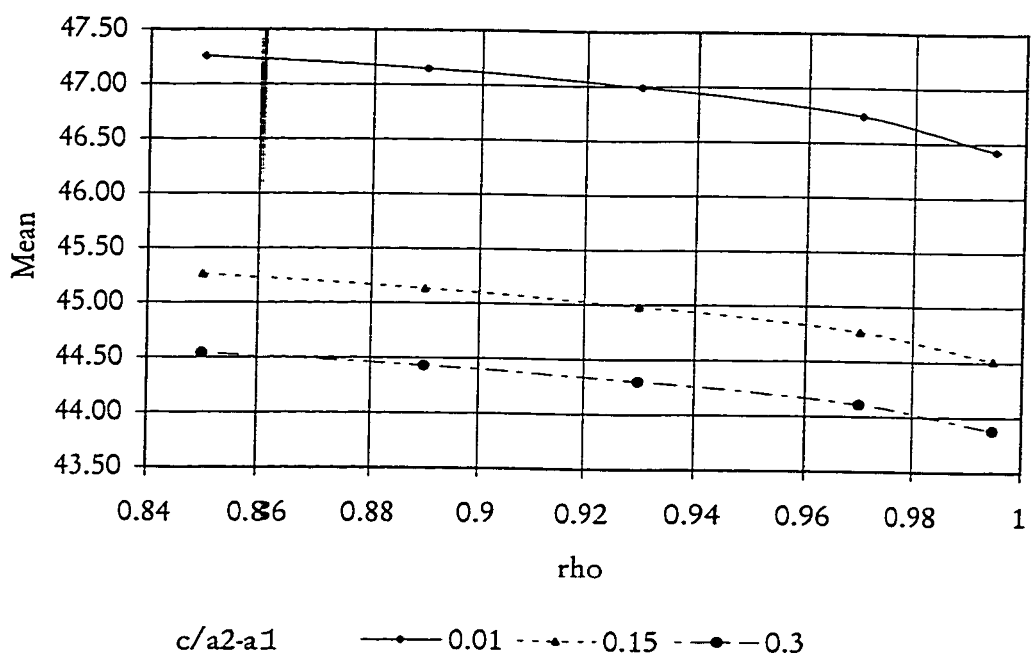


Figure 3-14: Mu versus (rho) at a2-r/a1-a2 = 8
sigma = 1.75

2. The optimal mean increase with the increase in $\frac{a_2 - r}{a_1 - a_2}$. This result is due to the reason that as a_2 gets closer to a_1 it will get more profitable (at any fixed cost) and the mean will set higher as a result.

3. The change in optimal mean is less with $\frac{a_2 - r}{a_1 - a_2}$ compared to $\frac{c}{a_1 - a_2}$. This is because cost is proportional to process mean while the selling prices are related to the probabilities of the product falling in respective grades. The effect of change in both the parameters will only be closer to each other if the units of the mean are proportional to the probabilities i.e., $\mu > 1$

4. The optimal mean decreases with the decrease in measurement error. As discussed above, the optimal process mean will be forced higher at higher error, as this will reduce the probability of the products falling near the specification limit.

3.3.2.3 EFFECT OF PARAMETERS ON CUT OFF POINTS

The effect on cut off points, by varying the parameters, is extremely important in evaluating the model performance. These points play a vital role in reducing the effect of loss in profit due to misclassification caused by measurement error. The value of the cut off point depends mostly upon the loss in profit (grade 1 sold as grade 2) and the penalty imposed if the lower grade classified as higher grade (warranty cost, loss of good will etc.) position of the mean (how close it is to the specification limit, which in turn will determine the probability of misclassification).

The optimal cut off values, are shown in Table 3-3. The penalties are taken at a fixed level and are set at a high value. The results show that both the cut of points are converging to the respective specification limit as $\rho \to 1$ see figures 3-15 to 3-20. As the mean is set quite higher than the L_2 the number of items misclassifying near L_2 will be significantly less. On the other hand, the chances of items misclassified around L_1 are substantially higher as compared to L_2 so the main emphasis is kept on w_1.

The figure shows that the cut off point w_1, initially increase then decrease, especially at lower costs, with the decrease in measurement error. This initial rise is because the optimal mean with lowering error gets closer to the specification limits (as shown in figures 3-9 to 3-14) and showing an increase in probability of misclassification. Consequently, with high penalties, the cut off point w_1 tend to increase initially, but as $\rho \to 1$ the error reduces as a result, the w_1 starts getting closer to the specification limit L_1 again. Another noticeable thing is that the w_1 starts initially lower than L_1 then crosses it over and finally starts bending again towards the specification limit L_1. The point, during this rise and fall, at which the value of w_1 becomes equal to the L_1 shows that, at that level of error the model manages to find a location where the expected loss of the misclassification on either side becomes the same.

The above discussion can be summarized as follows:

TABLE 3-3: Optimal Cut off points at different parameter settings

			$\frac{c}{a_1 - a_2}$					
			0.01		0.15		0.3	
σ_v	$\frac{a_2 - r}{a_1 - a_1}$	ρ	w_1	w_2	w_1	w_2	w_1	w_2
1.25	4	0.85	41.0079	37.9077	41.5647	38.4646	41.7920	38.6944
		0.89	41.2942	38.5918	41.6850	38.9821	41.8411	39.1393
		0.93	41.5135	39.1817	41.7515	39.4196	41.8439	39.5123
		0.97	41.6443	39.6868	41.7414	39.7838	41.7777	39.8202
		0.995	41.6128	39.9557	41.6283	39.9713	41.6339	39.9769
	6	0.85	41.1746	37.7738	41.7254	38.3237	41.9450	38.5454
		0.89	41.4335	38.4945	41.8196	38.8790	41.9685	39.0288
		0.93	41.6227	39.1153	41.8568	39.3492	41.9434	39.4362
		0.97	41.7148	39.6506	41.8094	39.7446	41.8427	39.7785
		0.995	41.6414	39.9425	41.6563	39.9575	41.6614	39.9627
	8	0.85	41.2472	37.7224	41.7955	38.2693	42.0115	38.4875
		0.89	41.4944	38.4582	41.8784	38.8393	42.0240	38.9859
		0.93	41.6705	39.0907	41.9027	39.3225	41.9867	39.4078
		0.97	41.7458	39.6372	41.8390	39.7299	41.8710	39.7624
		0.995	41.6540	39.9378	41.6686	39.9528	41.6734	39.9576
1.75	4	0.85	40.8651	37.3698	41.6554	38.1590	41.9616	38.4651
		0.89	41.2523	38.2311	41.8014	38.7797	42.0098	38.9879
		0.93	41.5443	38.9742	41.8748	39.3045	41.9977	39.4272
		0.97	41.7116	39.6088	41.8448	39.7417	41.8935	39.7907
		0.995	41.6592	39.9454	41.6803	39.9666	41.6879	39.9742
	6	0.85	41.1018	37.1806	41.8755	37.9519	42.1607	38.2367
		0.89	41.4479	38.0914	41.9826	38.6258	42.1739	38.8165
		0.93	41.6960	38.8799	42.0151	39.1985	42.1263	39.3102
		0.97	41.8091	39.5570	41.9361	39.6832	41.9794	39.7272
		0.995	41.6989	39.9268	41.7187	39.9467	41.7255	39.9534
	8	0.85	41.2043	37.1083	41.9704	37.8710	42.2468	38.1468
		0.89	41.5331	38.0386	42.0611	38.5661	42.2454	38.7496
		0.93	41.7624	38.8447	42.0761	39.1578	42.1826	39.2650
		0.97	41.8519	39.5380	41.9758	39.6611	42.0171	39.7033
		0.995	41.7163	39.9201	41.7356	39.9395	41.7420	39.9459

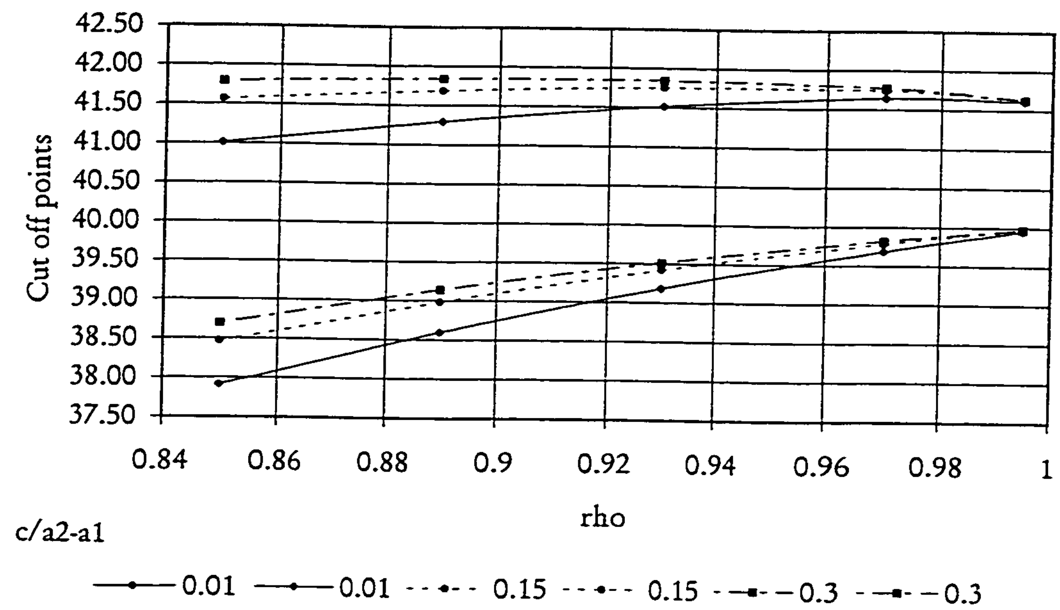


Figure 3-15: w1, w2 versus (rho) at a2-r/a1-a2 = 4
sigma = 1.25

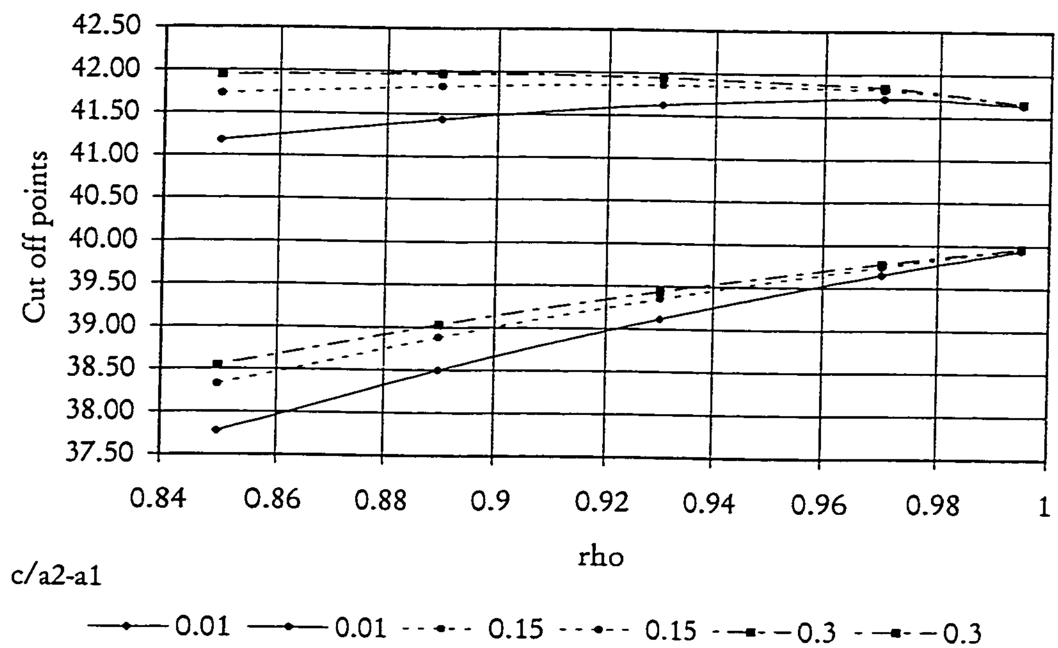


Figure 3-16: w1, w2 versus (rho) at a2-r/a1-a2 = 6.5
sigma = 1.25

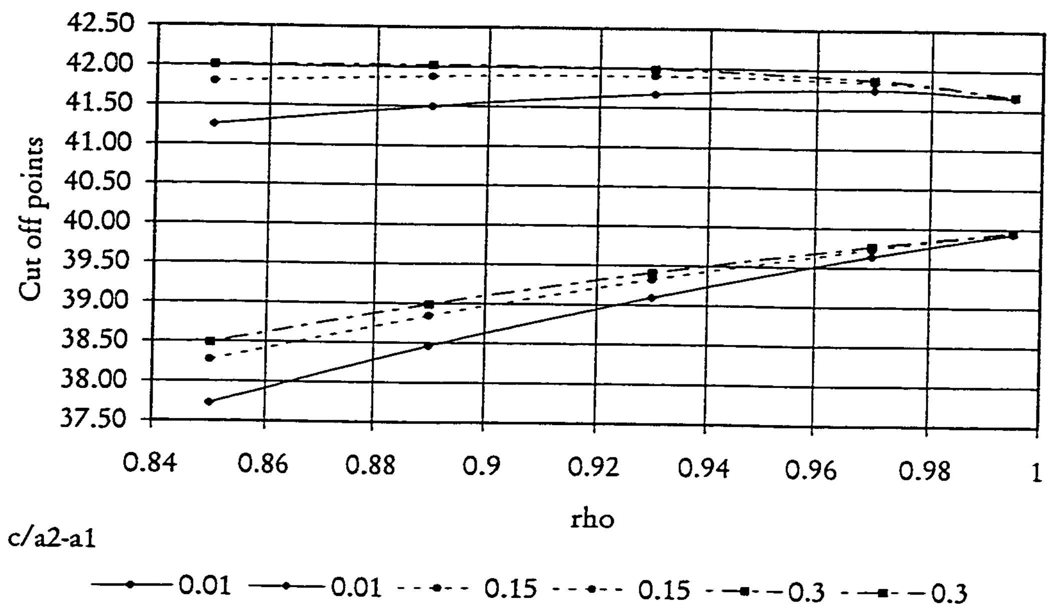


Figure 3-17: w1, w2 versus (rho) at a2-r/a1-a2 = 8
sigma = 1.25

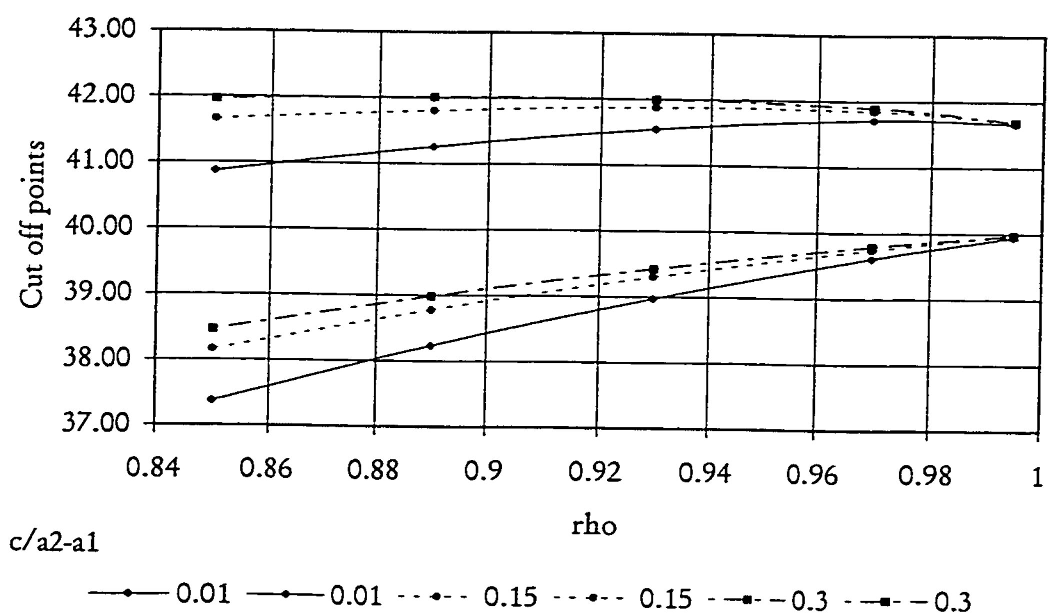


Figure 3-18: w1, w2 versus (rho) at a2-r/a1-a2 = 4
sigma = 1.75

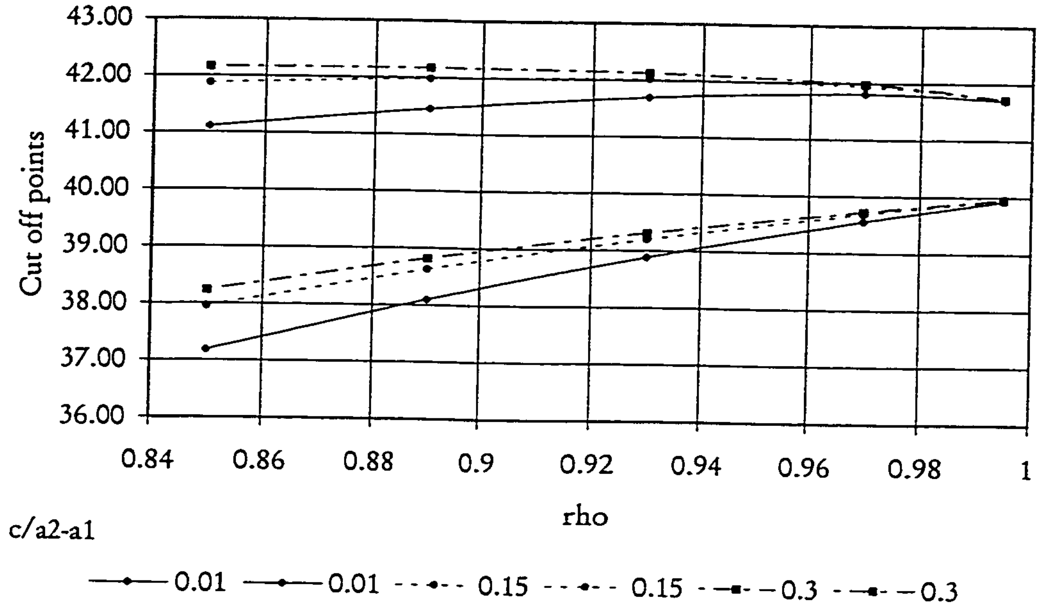


Figure 3-19: w1, w2 versus (rho) at a2-r/a1-a2 = 6.5
sigma = 1.75

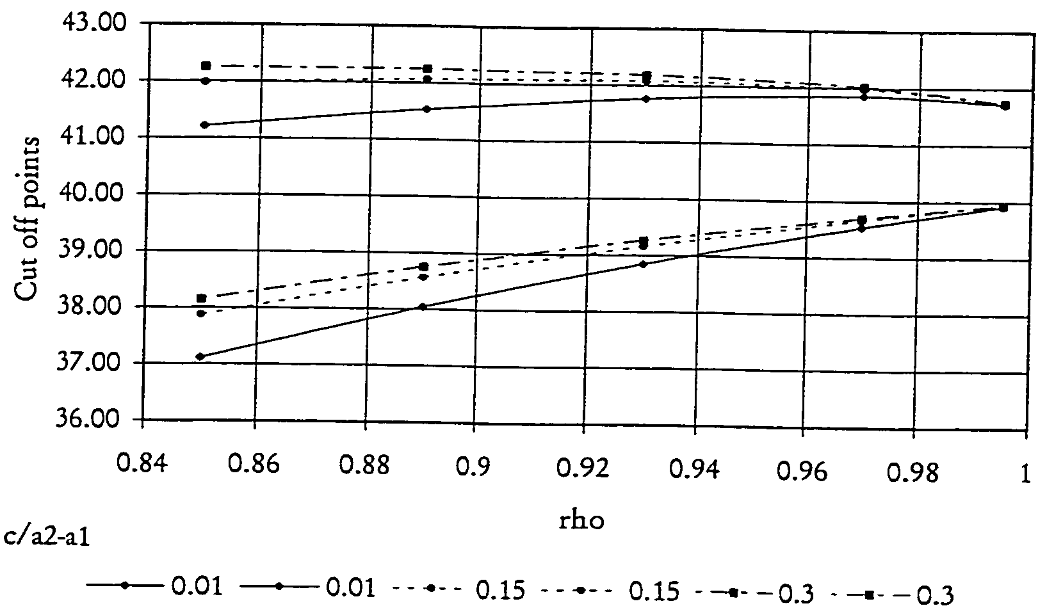


Figure 3-20: w1, w2 versus (rho) at a2-r/a1-a2 = 8
sigma = 1.75

1. Both the cut of points are converging to the respective specification limit as $\rho \to 1$.
2. The changes in cut off points with measurement error are higher at lower cost because of rapid changes in mean at lower costs (as shown in figures 3-9 to 3-14).
3. The variation in w_2 is higher than w_1. This is because the mean is far from w_2 and the effect of changing, w_2 is less as compared to w_1.
4. At higher level of costs, the results tend to remain conservative, (both cut off point tend to remain higher) i.e., it is better to lose profit than paying the penalty.

3.4 CONCLUSION

In this chapter, a model for multi-class screening targeting problem is developed for the case of measurement systems with error. The sensitivity analysis was performed and the outcome shows good results. Model performed very well in rectifying the effect of error. The rest of the parameters have the effects as expected.

CHAPTER 4

PROCESS TARGETING WITH MULTI-CLASS SCREENING AND UNIFORMITY PENALTY

4.1 INTRODUCTION

The purpose of this chapter is to extend model 1, presented in chapter 2, to the case where uniformity of the product, besides conformity, is considered. To achieve consistency, a penalty similar to the Taguchi quadratic loss function will be used as shown in figure 4-1. Here the function is assumed symmetric, however it is more realistic if a non-symmetric function is used i.e., 'K' different on either side of the target. The target is assumed set at 't' by the society. Each item will be penalized for inconsistency if the quality characteristic 'Y' is found off a fixed target 't'. As scrap is assumed not to be exposed to the society, no penalty for uniformity is associated in this grade. The objective of this model is to maximize the expected profit and reducing the effect of non-uniformity on the profit by finding the optimal process mean.

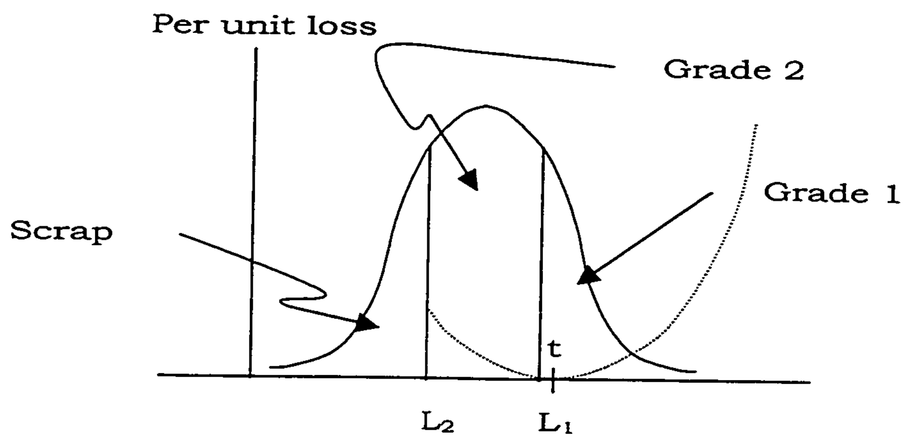


Figure 4-1: A production process with multi class screening & quadratic penalty for uniformity (dotted line)

This chapter is organized as follows: in section 4.2, the model development will be discussed, in section 4.2.3 two special cases are discussed, derived form Model 3 (EPM3). In the first case, the target value of the process is assumed to lie on the lower specification limit of the grade 1 i.e., L_1. In the second case, the target value of the process is assumed to be at the mean of the process, this section is followed by the results and conclusion.

4.2 MODEL DEVELOPMENT

In this section, the new model developed for multi-class screening targeting model under uniformity penalty will be presented. In section 4.2.1, the model assumptions will be presented followed by the model developed in section 4.2.2. Finally, in section 4.2.3 the two special cases will be discussed.

4.2.1 MODEL ASSUMPTIONS

The assumptions of the model are:

1. A single item is to be sold in two different markets with different cost/profit structures.
2. The inspection process is error free.
3. The quality characteristic 'Y' is assumed normally distributed with unknown process mean μ_Y and known variance σ_Y^2.
4. The inspection is based on 'Y'.
5. $a_1 > a_2 > r$.
6. $L_1 > L_2$
7. The production cost per item is $c_0 + cy$.

Specification limits on different grade are:

Specification limits on 'Y' for grade 1 are $Y \geq L_1$,

Specification limits on 'Y' for grade 2 are $L_2 \leq Y < L_1$,

Specification limits on 'Y' for scrap are $Y < L_2$,

4.2.2 STATEMENT OF THE MODEL

Now the profit function P(y) can be expressed as

$$P(y) = \begin{cases} a_1 - (c_0 + cy + c_i) - K(y-t)^2 & Y \geq L_1 \\ a_2 - (c_0 + cy + c_i) - K(y-t)^2 & L_2 \leq Y < L_1 \\ r - (c_0 + cy + c_i) & Y < L_2 \end{cases}$$

As compared to model 1, the first two terms contain additional term each for incorporating the effect of inconsistency. The 'K' is defined per squared deviation penalty per item. The term $(y - t)^2$ determines the squared deviation in the product characteristic. If the quality characteristic 'Y' lies exactly on target 't' this deviation term vanishes and there will be no penalty for inconsistency.

$\therefore$The expected profit per item can be written as

$$E\{P(\mu,t)\} = \int_{L_1}^{\infty}[a_1 - (c_0 + cy + c_i) - K(y-t)^2]\varphi\, dy$$

$$+ \int_{L_2}^{L_1}[a_2 - (c_0 + cy + c_i) - K(y-t)^2]\varphi\, dy$$

$$+ \int_{-\infty}^{L_2}[r - (c_0 + cy + c_i)]\varphi\, dy$$

$$\Rightarrow \quad E\{P(\mu,t)\} = (a_1 - c_0 - c_i)\int_{L_1}^{\infty}\varphi\,dy + (a_2 - c_0 - c_i)\int_{L_2}^{L_1}\varphi\,dy + (r - c_0 - c_i)\int_{-\infty}^{L_2}\varphi\,dy - \int_{L_1}^{\infty}\left[K(y-t)^2\right]\varphi\,dy - \int_{L_2}^{L_1}\left[K(y-t)^2\right]\varphi\,dy - c\int_{-\infty}^{\infty} y\varphi\,dy$$

$$E\{P(\mu,t)\} = a_1\int_{L_1}^{\infty}\varphi\,dy + a_2\int_{L_2}^{L_1}\varphi\,dy + r\int_{-\infty}^{L_2}\varphi\,dy - (c_0 + c_i)\int_{-\infty}^{\infty}\varphi\,dy - c\int_{-\infty}^{\infty} y\varphi\,dy - \int_{L_2}^{\infty}\left[K(y-t)^2\right]\varphi\,dy$$

$$E\{P(\mu,t)\} = \left[a_1\int_{L_1}^{\infty}\varphi\,dy + a_2\int_{L_2}^{L_1}\varphi\,dy + r\int_{-\infty}^{L_2}\varphi\,dy - (c_0 + c_i) - c\mu\right] - \int_{L_2}^{\infty}\left[K(y-t)^2\right]\varphi\,dy \qquad (4\text{-}1)$$

In equation (4-1) the first four terms in the initial bracket is the profit function of the model presented by Min & Lee 1997 i.e., MODEL 1 (EPM1) (equation (3-3)). Therefore, the above equation can be restated as:

$$E\{P(\mu,t)\} = EPM1 - \int_{L_2}^{\infty}\left[K(y-t)^2\right]\varphi\,dy \qquad (4\text{-}2)$$

Let,

$$(y-t)^2 = [(y-\mu)+(\mu-t)]^2 = (y-\mu)^2 + 2(y-\mu)(\mu-t) + (\mu-t)^2$$

Using above relationship the profit function becomes:

$$E\{P(\mu,t)\} = EPM1$$
$$-K\left[\int_{L_2}^{\infty}(y-\mu)^2\varphi\,dy + 2(\mu-t)\int_{L_2}^{\infty}(y-\mu)\varphi\,dy + (\mu-t)^2\int_{L_2}^{\infty}\varphi\,dy\right] \quad (4\text{-}3)$$

Let,

$$z = \frac{y-\mu}{\sigma_y}$$

$$\Rightarrow \qquad y = \mu + z\sigma$$

The equation (4-3) can be restated as:

$$E\{P(\mu,t)\} = EPM1$$
$$-K\left[\int_{\Gamma_2}^{\infty}(\sigma z)^2\varphi(z)dz + 2(\mu-t)\int_{\Gamma_2}^{\infty}(\sigma z)\varphi(z)dz + (\mu-t)^2\int_{\Gamma_2}^{\infty}\varphi(z)dz\right]$$

or

$$EPM3 = EPM1$$
$$-K\left[\varphi(\Gamma_2)\left[\sigma^2 z\varphi(\Gamma_2) + 2\sigma(\mu-t)\right] + \left[\sigma_Y^2 + (\mu-t)^2\right]\left[1-\Phi(\Gamma_2)\right]\right] \quad (4\text{-}4)$$

4.2.3 SPECIAL CASES

In this section, two special cases will be derived from Model 3 (EPM3). In case I, the process target is at the mean of the process while in the case II the target is set at L_1.

4.2.3.1 SPECIAL CASE I {MODEL 3-I ($EPM3_I$)}

In this case, the target is assumed to be at the mean of the process.

$$t = \mu \text{, (see figure 4-2(a))}$$

Letting $E\{P(\mu)\} \equiv EPM3_I$ we can rewrite the equation (4-4) as:

$$\begin{aligned} EPM3 = EPM1 \\ - K\left[\varphi(\Gamma_2)\left[\sigma^2 z\varphi(\Gamma_2) + 2\sigma(\mu-\mu)\right] + \left[\sigma_Y^2 + (\mu-\mu)^2\right]\left[1-\Phi(\Gamma_2)\right]\right] \end{aligned}$$

or

$$\begin{aligned} EPM3 = EPM1 \\ - K\left[\varphi(\Gamma_2)\left[\sigma^2 z\varphi(\Gamma_2)\right] + \sigma_Y^2\left[1-\Phi(\Gamma_2)\right]\right] \end{aligned}$$

4.2.3.2 SPECIAL CASE II {MODEL 3-II ($EPM3_{II}$)}

In this case, the target for uniformity i.e., 't' is assumed to be set at the specification limit of the grade 1 (see figure 4-2(b)) i.e.,

$$t = L_1$$

Letting $E\{P(\mu)\} \equiv EPM3_{II}$ we can rewrite the equation (4-4) as:

$$\begin{aligned} EPM3 = EPM1 \\ - K\left[\varphi(\Gamma_2)\left[\sigma^2 z\varphi(\Gamma_2) + 2\sigma(\mu-L_1)\right] + \left[\sigma_Y^2 + (\mu-L_1)^2\right]\left[1-\Phi(\Gamma_2)\right]\right] \end{aligned}$$

4.3 RESULTS AND ANALYSIS

In this section, an illustrative example will be presented. This will be followed by the sensitivity analysis. For the numerical analysis, Mathematica 4.0 is used. The analysis is performed on a Compaq Deskpro, Pentium III computer with 64 MB of RAM. See appendix B for the notebook (program code).

4.3.1 ILLUSTRATIVE EXAMPLE

Consider a packing plant of a cement factory. The plant consists of two processes a filling process and an inspection process. Each cement bag processed by filling machine is moved to the loading

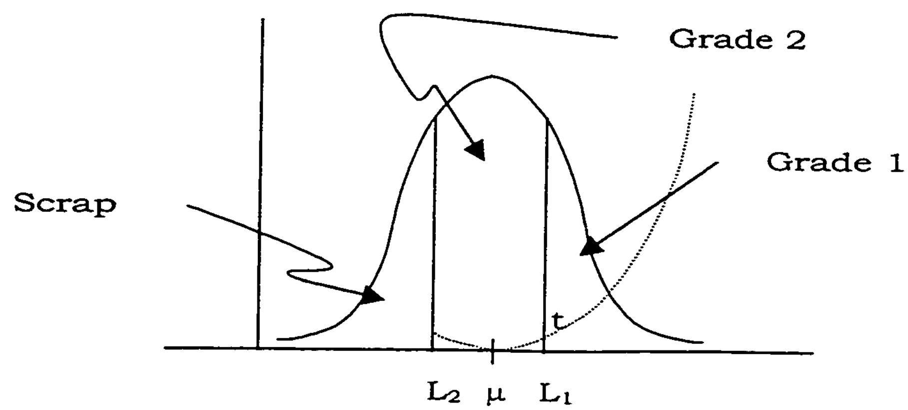


Figure 4-2(a): Special case I (EPM3$_I$) (target at μ)

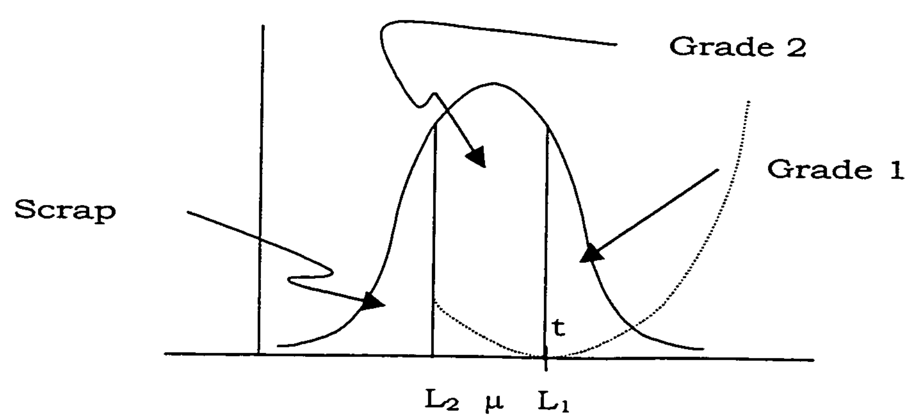


Figure 4-2(b): Special case II (EPM3$_{II}$) (target as L$_1$)

and dispatching phase on the conveyor belt. Inspection is performed by automatic weighing system and is assumed error free. Suppose that the cost components and the specification limits are a_1=\$5.5, a_2=\$4.9, r=\$2.5, c_0=\$0.1, c_i=\$0.04, c=\$0.12, L_1=41.5 kg, L_2=40.0 kg, k=0.25 and $\sigma_y^2 = (1.5)^2$.

The expected profit and the optimal values of the mean and the cut off values are found out to be:

Model 1:

E(p) = \$1.28671/unit

μ = 44.3291 kg

Case I: target at μ

E(p) = \$1.10853/unit

μ = 44.09342 kg

Consider the case where the penalty for non uniformity was neglected, the gain in expected profit will be reflected if the mean obtained from model 1 is substituted in model obtained in case I i.e., E(p) = \$1.10635/unit

Gain in profit: \$0.00218/unit or approximately 0.197% gain per a_1. Now considering a million units produced per annum by a manufacturer, the net gain in expected profit per year is \$ 2180.

Case II: target at L_1

E(p) = \$0.90272/unit

As can be seen from figure 4-3 (c) the mean is forced very close to the target L_1 in order to have the maximum number of item close to the target L_1.

Consider the case where the penalty for non uniformity was neglected, the gain in expected profit will be reflected if the mean ob-

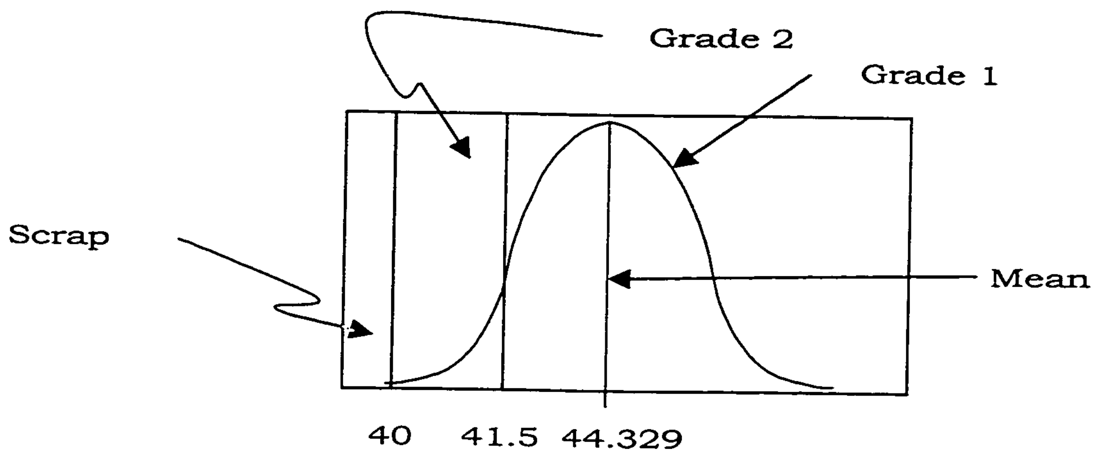


Figure 4-3 (a): Optimal mean settings obtained from model 1

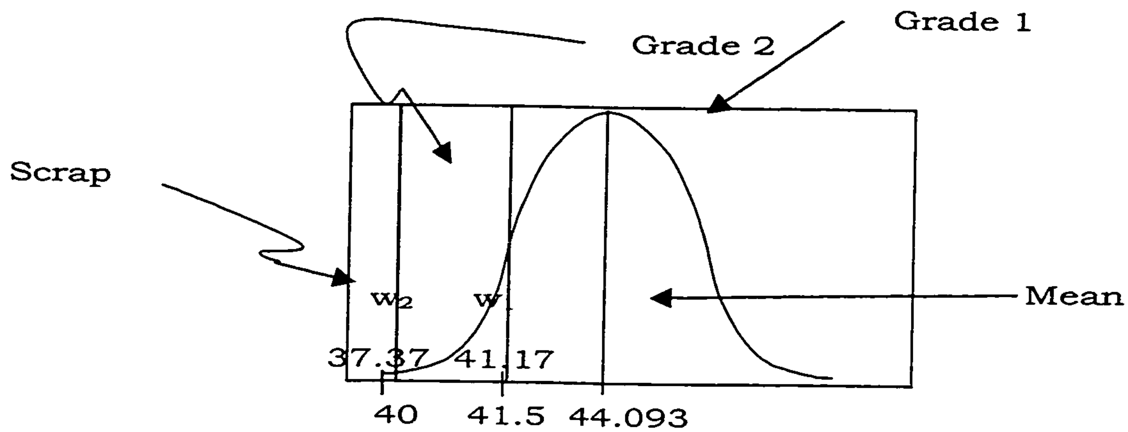


Figure 4-3 (b): Optimal mean obtained from model 3 case I

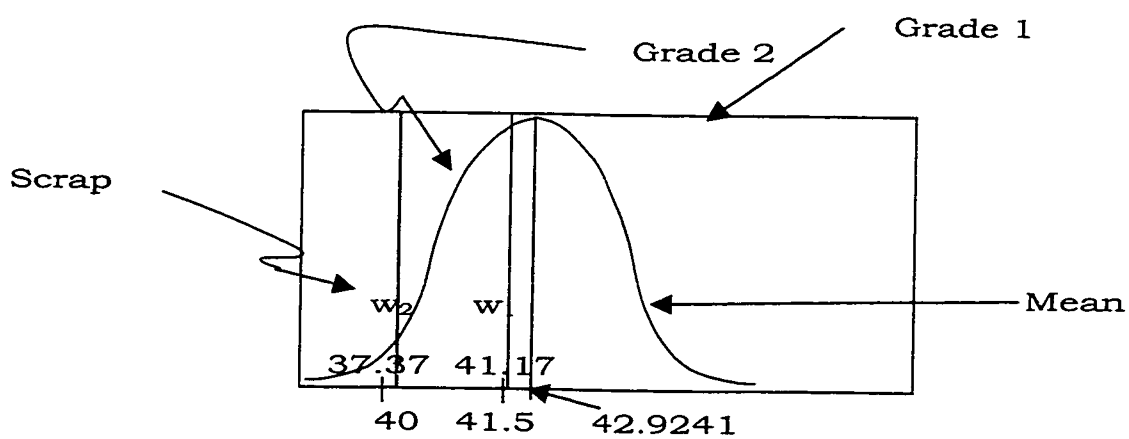


Figure 4-3 (C): Optimal mean obtained from model 3 case II

tained from model 1 is substituted in the model obtained in case II i.e.,

$E(p) = \$0.69055/\text{unit}$

Gain in profit: \$0.21217/unit or approximately 23.50% gain per a_1. Now considering a million units produced per annum by a manufacturer, the net gain in expected profit per year is \$212170, which, as can be seen, is a huge gain. In other words setting the mean higher for selling more products will result in a loss of \$212170 in terms of inconsistency penalties.

4.3.2 SENSITIVITY ANALYSIS

In this parametric analysis, the effect of different parameters on the output values i.e., the expected profit, the optimal mean are studied. There are two special cases derived in the preceding section. Both of these cases have been evaluated in this sensitivity analysis.

Before going to the to numerical study, a look at the rates of change of profit with different parameters will give better insight of the model behavior. The partial derivatives of the objective functions with the selling prices 'a_1', 'a_2' & 'r' and cost 'c' are as follows:

$$\frac{\partial EPM3}{\partial a_1} = \Phi(-\Gamma_1)$$

$$\frac{\partial EPM3}{\partial a_2} = \Phi(\Gamma_1) - \Phi(\Gamma_2)$$

$$\frac{\partial EPM3}{\partial r} = \Phi(\Gamma_2)$$

$$\frac{\partial EPM3}{\partial c} = -\mu$$

As can be seen above, the rates of change expected profit with selling prices are simply the probability of an item falling into that grade, while the rate of change of expected profit with cost is equal to the negative of the mean of the profit. It is clear that the effect of the cost on the expected profit is dominant. The plots of the rates of changes at different variance levels are plotted in figures 4-4(a) to 4-4(c). The specifcation limit are taken to be L_1=41.5 and L_2=40. The plots show that rate of change varies more rapidly with mean with lower variance which is obvious as the spread for the distribution is smaller i.e., the effect of changing mean with lower variances will have greater effect on profit than the case where the variances are higher.

For the sensitivity analysis, As there are a number of parameters, a few of them were chosen. The parameters are $\frac{c}{a_1 - a_2}$, $\frac{a_2 - r}{a_1 - a_2}$ and σ_y. These are the parameters used in Min Koo Lee & Joon Soong Jang (1997). Here $\frac{c}{a_1 - a_2}$ represents the 'per unit production cost' in dimensionless from, $\frac{a_2 - r}{a_1 - a_2}$ represents per item selling prices in the same manner, $(\sigma_y)^2$ is the variance of the process. The first two parameters are taken at eight different levels, while the third i.e., σ_y is set at four different levels.

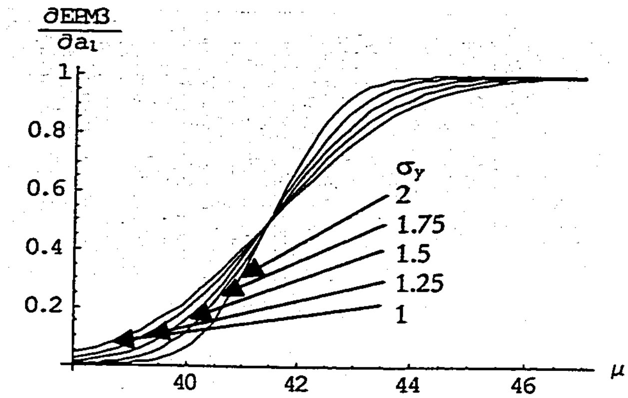


Figure 4-4(a): Rate of change of expected profit with a_1

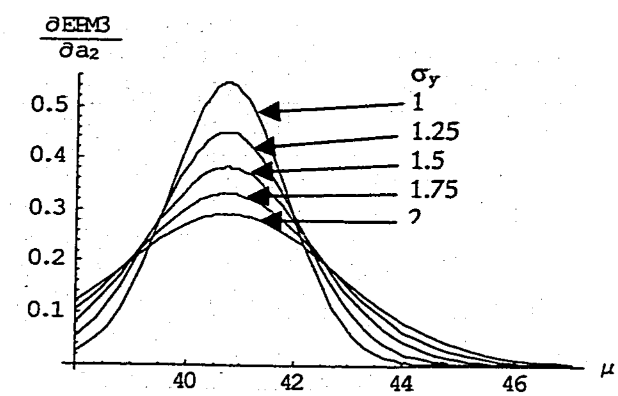


Figure 4-4(b): Rate of change of expected profit with a_2

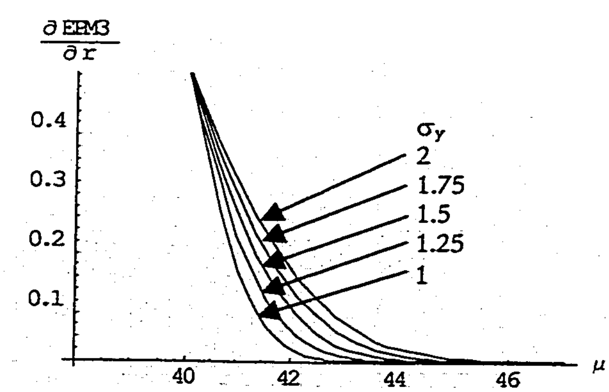


Figure 4-4(c): Rate of change of expected profit with r

4.3.2.1 EFFECTS ON EXPECTED PROFIT (SPECIAL CASE I)

The effects on expected profit by varying different parameters are presented in table 4-1. As can be seen, the result shows a higher variation with the cost than in selling price or σ_y. The results are shown graphically in figures 4-5 to 4-12. In figures 4-5 to 4-8 the expected profit is plotted against $\frac{a_2 - r}{a_1 - a_2}$ the result show a gain in expected profit with increasing $\frac{a_2 - r}{a_1 - a_2}$. This gain, as compared to selling price, is higher with lowering costs. Figures 4-9 to 4-12 shows the variation in expected profit with $\frac{c}{a_1 - a_2}$. The results are summarized as follows:

1. The expected profit decreases considerably with the increase in $\frac{c}{a_1 - a_2}$. This is due to term $(-c\mu)$ present in the model showing the proportional increase in the production cost per item with the increase in cost. As $\frac{c}{a_1 - a_2}$ increases the mean of the process decrease in order to reduce the cost and in turn reduces the expected profit.

2. The effect of increasing $\frac{a_2 - r}{a_1 - a_2}$ shows an expected gain in profit. The reason is simple i.e., as the selling price of a_2 becomes closer a_1 at a given cost the mean can be set higher resulting in profit increase.

TABLE 4-1: Expected profit at different parameter settings

(Special case I)

		$\frac{c}{a_1 - a_2}$						
σ_y	$\frac{a_2 - r}{a_1 - a_2}$	0.01	0.05	0.1	0.15	0.2	0.25	0.3
1.5	4.0	4.97219	3.89749	2.57393	1.26242	-0.04056	-1.33671	-2.62709
	5.0	5.01804	4.12199	3.01800	1.92363	0.83604	-0.24620	-1.32393
	5.5	5.03638	4.21181	3.19568	2.18824	1.18690	0.19037	-0.80214
	6.0	5.05013	4.27918	3.32896	2.38676	1.45015	0.51794	-0.41058
	6.5	5.06389	4.34655	3.46227	2.58533	1.71349	0.84567	-0.01881
	7.0	5.07764	4.41393	3.59561	2.78396	1.97694	1.17355	0.37318
	7.5	5.08681	4.45886	3.68452	2.91642	2.15263	1.39223	0.63463
	8.0	5.09599	4.50379	3.77344	3.04891	2.32838	1.61099	0.89620
1.75	4.0	4.92776	3.84213	2.50783	1.18729	-0.12347	-1.42641	-2.72269
	5.0	4.97423	4.06882	2.95538	1.85289	0.75812	-0.33052	-1.41399
	5.5	4.99281	4.15952	3.13450	2.11935	1.11113	0.10840	-0.88971
	6.0	5.00675	4.22756	3.26888	2.31929	1.37605	0.43783	-0.49617
	6.5	5.02070	4.29561	3.40331	2.51931	1.64111	0.76747	-0.10233
	7.0	5.03464	4.36367	3.53778	2.71943	1.90633	1.09736	0.29184
	7.5	5.04393	4.40906	3.62746	2.85290	2.08324	1.31742	0.55482
	8.0	5.05323	4.45445	3.71716	2.98642	2.26023	1.53760	0.81795
2	4.0	4.87734	3.78154	2.43731	1.10853	-0.20924	-1.51813	-2.81942
	5.0	4.92439	4.01028	2.88808	1.77813	0.67686	-0.41750	-1.50599
	5.5	4.94321	4.10181	3.06854	2.04629	1.03182	0.02353	-0.97952
	6.0	4.95733	4.17048	3.20395	2.24754	1.29826	0.35463	-0.58422
	6.5	4.97145	4.23917	3.33942	2.44891	1.56490	0.68603	-0.18851
	7.0	4.98557	4.30788	3.47495	2.65042	1.83176	1.01774	0.20763
	7.5	4.99498	4.35369	3.56535	2.78484	2.00979	1.23907	0.47198
	8.0	5.00440	4.39952	3.65579	2.91932	2.18794	1.46056	0.73654
2.25	4.0	4.82104	3.71611	2.36296	1.02686	-0.29700	-1.61092	-2.91627
	5.0	4.86866	3.94674	2.81666	1.70005	0.59303	-0.50627	-1.59898
	5.5	4.88770	4.03904	2.99834	1.96974	0.94973	-0.06340	-1.07066
	6.0	4.90199	4.10829	3.13468	2.17217	1.21753	0.26917	-0.67385
	6.5	4.91628	4.17757	3.27111	2.37477	1.48559	0.60211	-0.27653
	7.0	4.93056	4.24687	3.40762	2.57754	1.75392	0.93545	0.12133
	7.5	4.94009	4.29309	3.49868	2.71282	1.93297	1.15791	0.38688
	8.0	4.94961	4.33932	3.58979	2.84818	2.11216	1.38057	0.65271

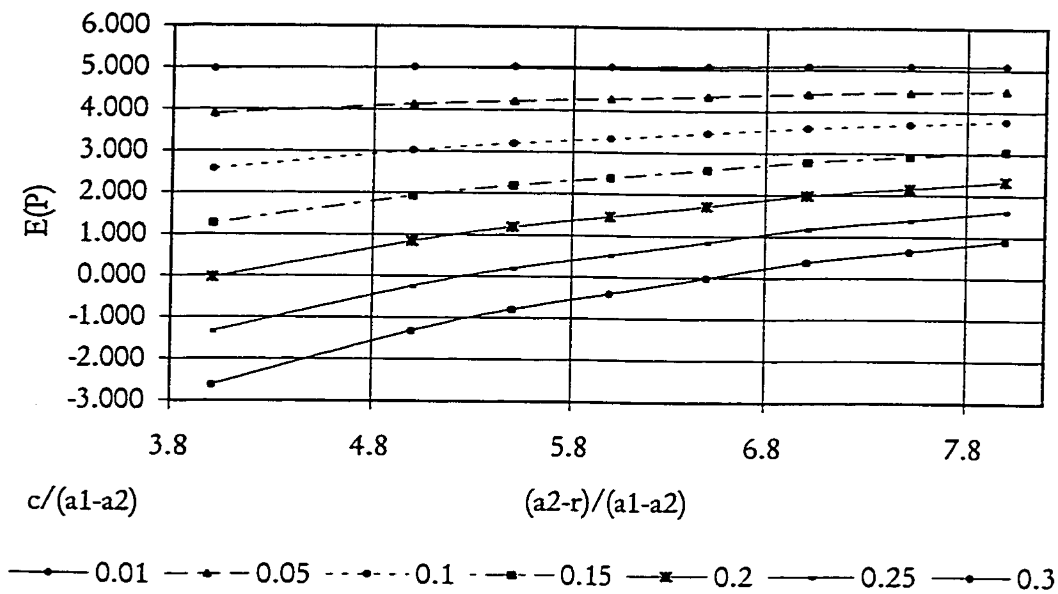


Figure 4-5: E(p) versus a2-r/a1-a2 at sigma = 1.5
Special case I

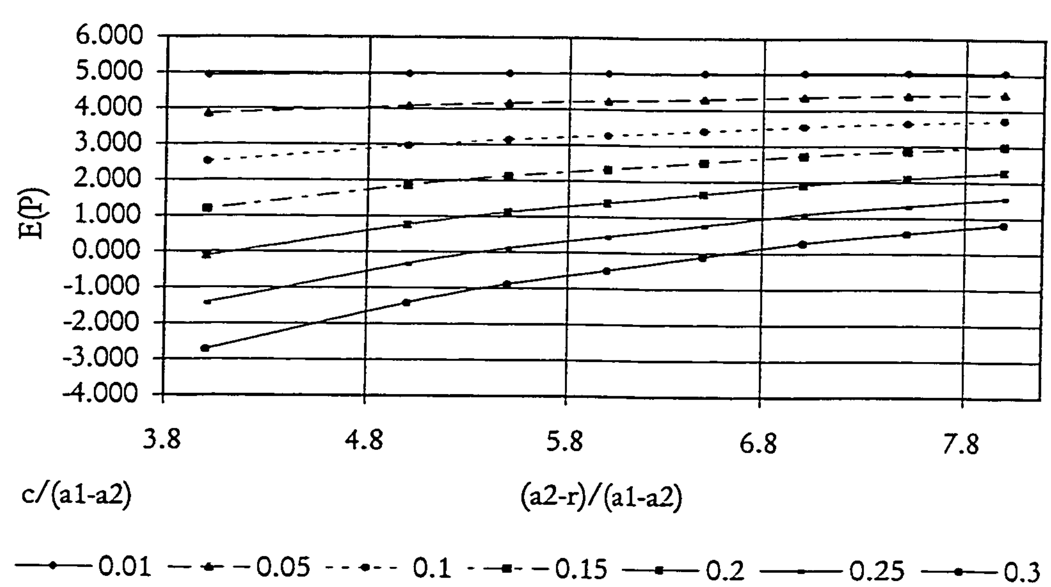


Figure 4-6: E(p) versus a2-r/a1-a2 at sigma = 1.75
Special case I

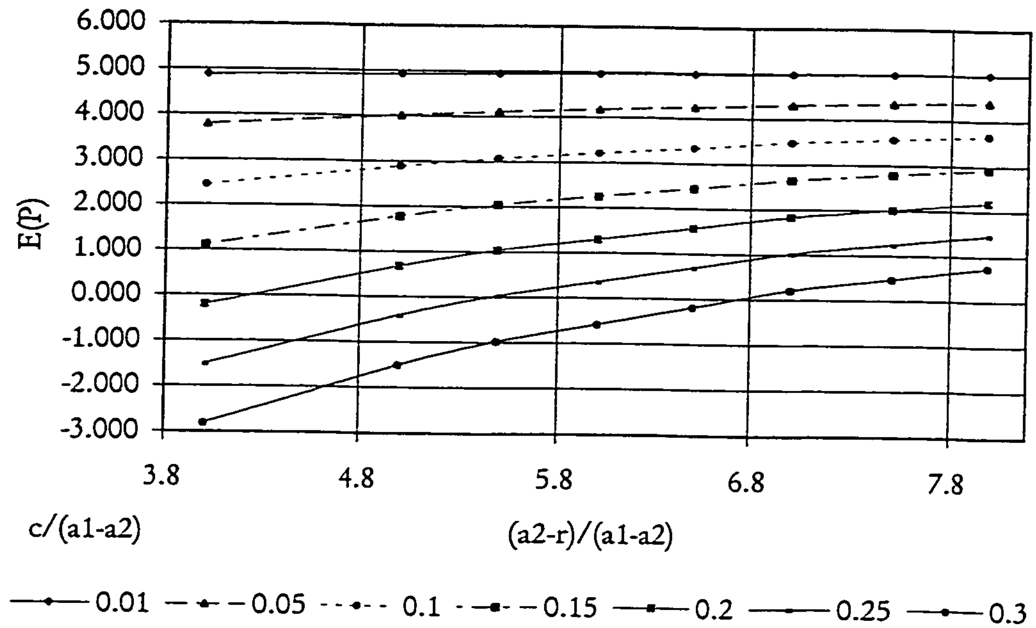


Figure 4-7: E(p) versus a2-r/a1-a2 at sigma = 2
Special case I

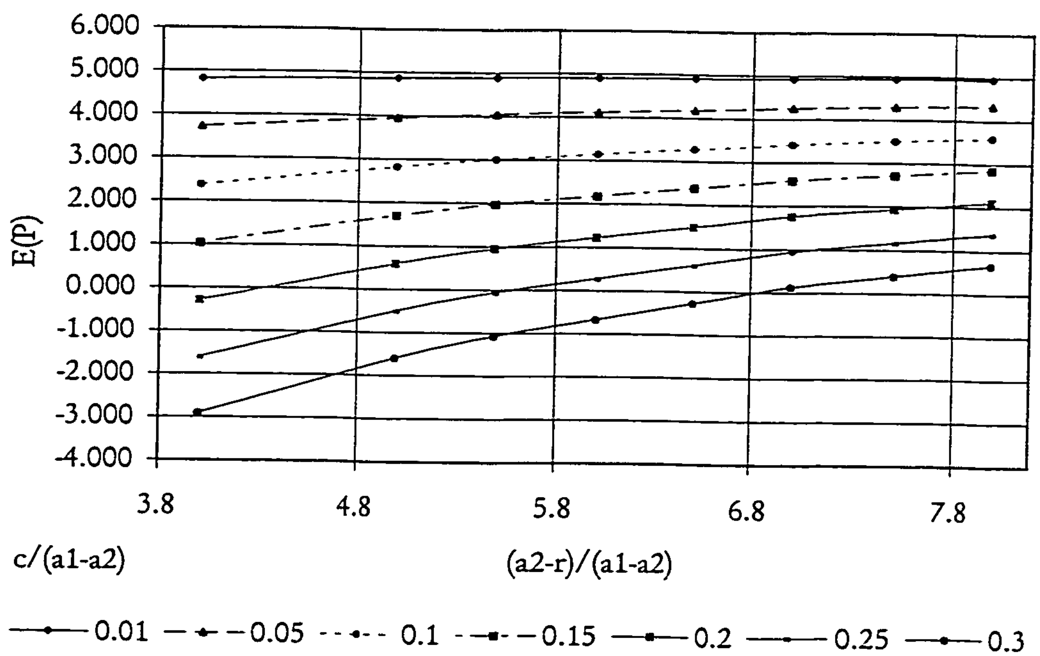


Figure 4-8: E(p) versus a2-r/a1-a2 at sigma = 2.25
Special case I

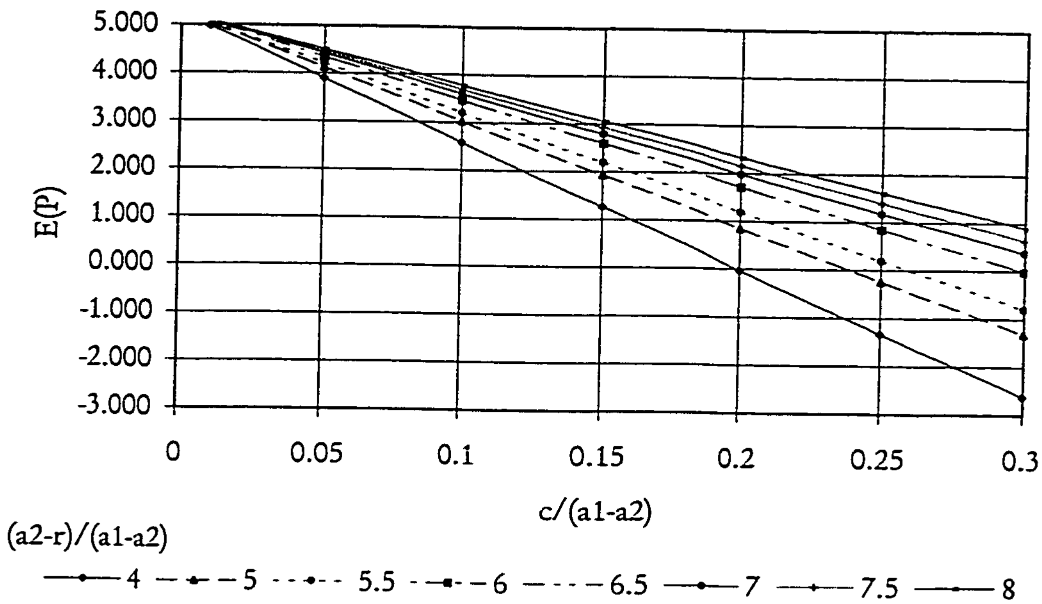


Figure 4-9: E(p) versus c/a1-a2 at sigma = 1.5
Special case I

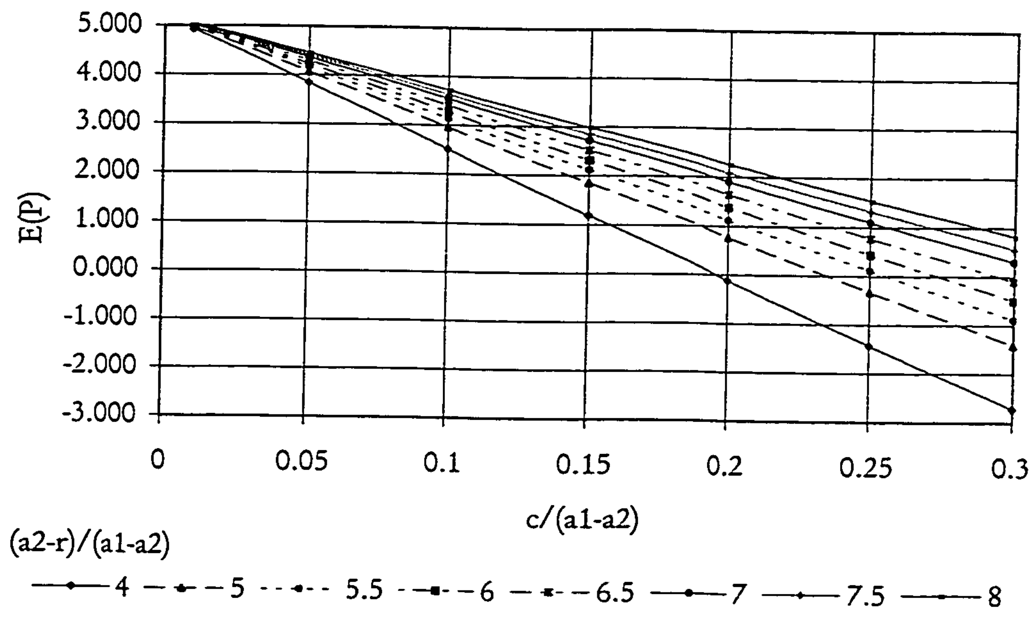


Figure 4-10: E(p) versus c/a1-a2 at sigma = 1.75
Special case I

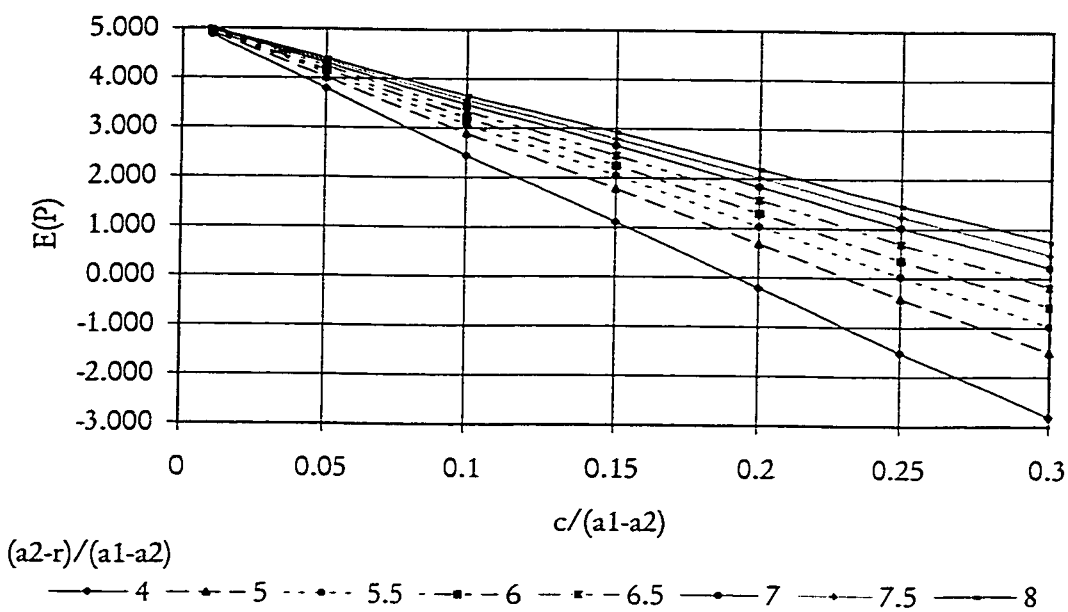


Figure 4-11: E(p) versus c/a1-a2 at sigma = 2
Special case I

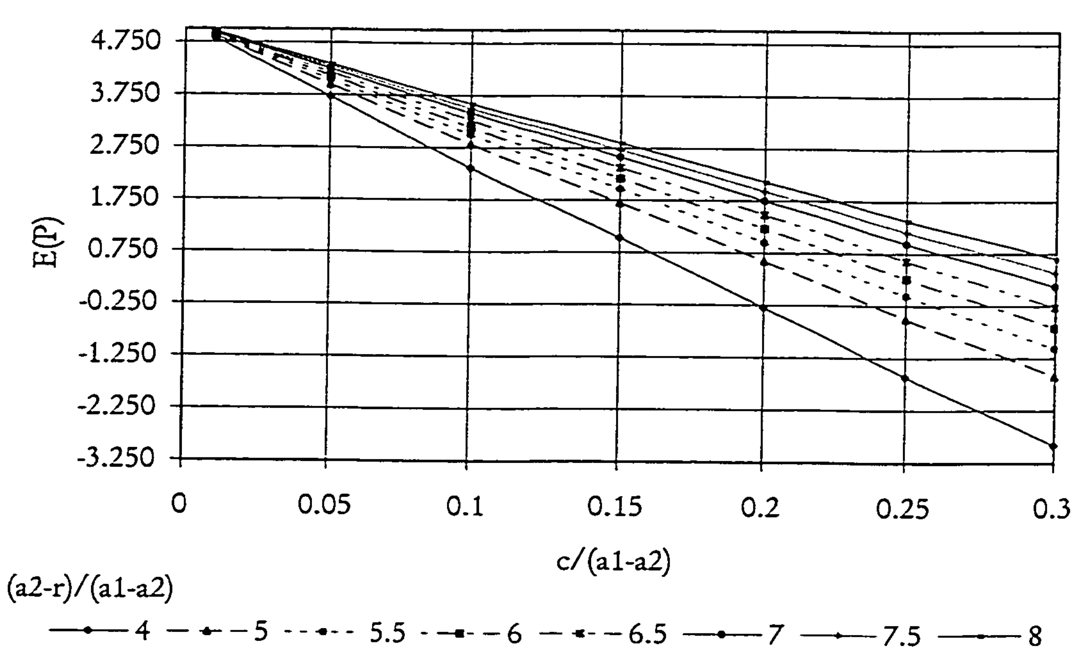


Figure 4-12: E(p) versus c/a1-a2 at sigma = 2.25
Special case I

The effect of $\frac{c}{a_1 - a_2}$ is relatively higher as compared to $\frac{a_2 - r}{a_1 - a_2}$. The reason behind it evident from the model. The rate of gain in profit is in terms of probability of the product falling in that grade, while the cost is directly related to the process mean. This effect will decrease as the units of measurement becomes proportional to the probabilities i.e., the unit of measurement is > 1

4.3.2.2 EFFECTS ON OPTIMAL MEAN (SPECIAL CASE I)

The effects on optimal mean by varying different parameters are presented in table 4-2. The result shows that the effect of cost is higher on the optimal mean than the selling price. The results are shown graphically in figures 4-13 to 4-20. In figures 4-13 to 4-16 the optimal is plotted against $\frac{a_2 - r}{a_1 - a_2}$ the result show an increase in optimal mean with the increase in $\frac{a_2 - r}{a_1 - a_2}$. In figures, 4-17 to 4-20 the change in optimal mean with the change in cost is plotted. The results are summarized below:

1. The optimal mean increases with the increase in $\frac{a_2 - r}{a_1 - a_2}$. This is because as the selling price of grade increases (keeping the other fixed) the optimal mean will be set higher.

The change is optimal mean with cost i.e., $\frac{c}{a_1 - a_2}$ is significant as compared to $\frac{a_2 - r}{a_1 - a_2}$. There is sharp decrease in mean with the increase in mean. This is an expected result due to the di

TABLE 4-2: Optimal Mean at different parameter settings

(Special case I)

σ_y	$\frac{a_2 - r}{a_1 - a_2}$	$\frac{c}{a_1 - a_2}$: 0.01	0.05	0.1	0.15	0.2	0.25	0.3
	4.0	45.38295	44.39490	43.88980	43.56234	43.31165	43.10432	42.92488
	5.0	45.39481	44.42711	43.93964	43.62696	43.38963	43.19487	43.02756
	5.5	45.40092	44.44273	43.96316	43.65689	43.42521	43.23566	43.07328
1.5	6.0	45.40596	44.45576	43.98251	43.68127	43.45396	43.26840	43.10977
	6.5	45.41168	44.47012	44.00357	43.70757	43.48477	43.30328	43.14844
	7.0	45.41805	44.48606	44.02662	43.73609	43.51795	43.34064	43.18966
	7.5	45.42307	44.49770	44.04325	43.75652	43.54159	43.36712	43.21876
	8.0	45.42896	44.51028	44.06106	43.77825	43.56660	43.39504	43.24933
	4.0	45.93242	44.79281	44.21508	43.84083	43.55364	43.31517	43.10771
	5.0	45.95101	44.84168	44.28708	43.93129	43.66046	43.43732	43.24470
	5.5	45.96040	44.86495	44.32050	43.97257	43.70856	43.49167	43.30500
1.75	6.0	45.96833	44.88417	44.34774	44.00592	43.74715	43.53503	43.35285
	6.5	45.97716	44.90516	44.37715	44.04164	43.78824	43.58096	43.40332
	7.0	45.98718	44.92823	44.40906	44.08011	43.83223	43.62988	43.45682
	7.5	45.99501	44.94495	44.43194	44.10751	43.86339	43.66439	43.49443
	8.0	46.00243	44.96288	44.45627	44.13648	43.89622	43.70063	43.53380
	4.0	46.44366	45.16217	44.51441	44.09342	43.76872	43.49749	43.25994
	5.0	46.46751	45.22717	44.60803	44.20924	43.90416	43.65140	43.43194
	5.5	46.47924	45.25766	44.65094	44.26152	43.96453	43.71923	43.50696
2	6.0	46.48909	45.28266	44.68568	44.30351	44.01271	43.77308	43.56619
	6.5	46.50013	45.30978	44.72296	44.34825	44.06378	43.82988	43.62840
	7.0	46.51236	45.33936	44.76317	44.39617	44.11817	43.89010	43.69407
	7.5	46.52136	45.36064	44.79183	44.43014	44.15655	43.93242	43.74006
	8.0	46.53112	45.38335	44.82219	44.46593	44.19685	43.97672	43.78807
	4.0	46.90043	45.49479	44.78201	44.31568	43.95348	43.64869	43.37964
	5.0	46.92569	45.57443	44.89613	44.45607	44.11712	43.83443	43.58736
	5.5	46.93792	45.61133	44.94790	44.51885	44.18943	43.91562	43.67717
2.25	6.0	46.94814	45.64138	44.98957	44.56904	44.24689	43.97978	43.74778
	6.5	46.95942	45.67378	45.03408	44.62229	44.30755	44.04721	43.82165
	7.0	46.97178	45.70888	45.08182	44.67906	44.37190	44.11841	43.89933
	7.5	46.98104	45.73401	45.11571	44.71914	44.41714	44.16829	43.95356
	8.0	46.99092	45.76069	45.15147	44.76124	44.46451	44.22035	44.01000

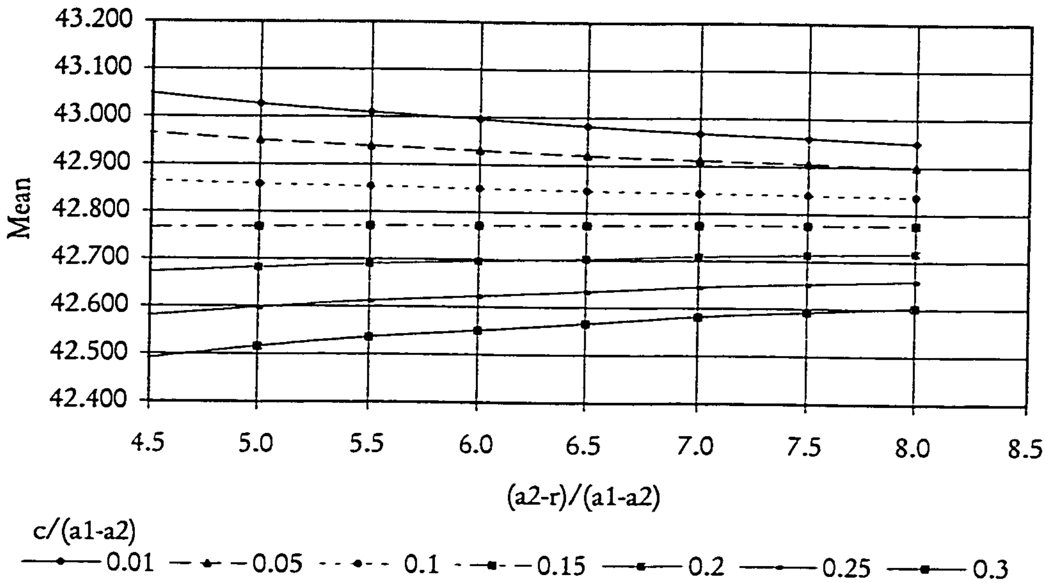


Figure 4-13: Mu versus a2-r/a1-a2 at sigma = 1.5
Special case I

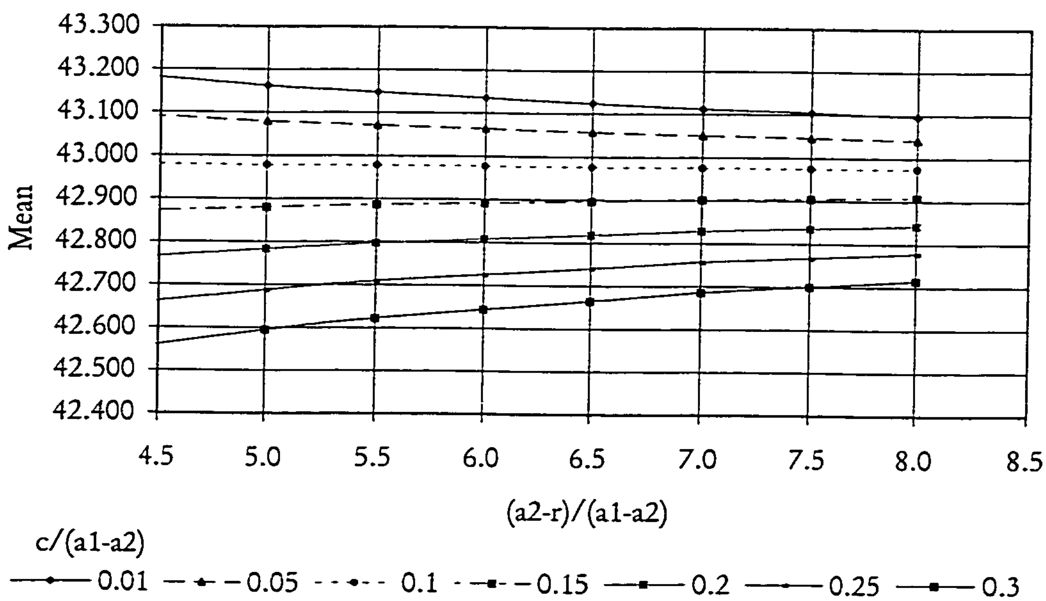


Figure 4-14: Mu versus a2-r/a1-a2 at sigma = 1.75
Special case I

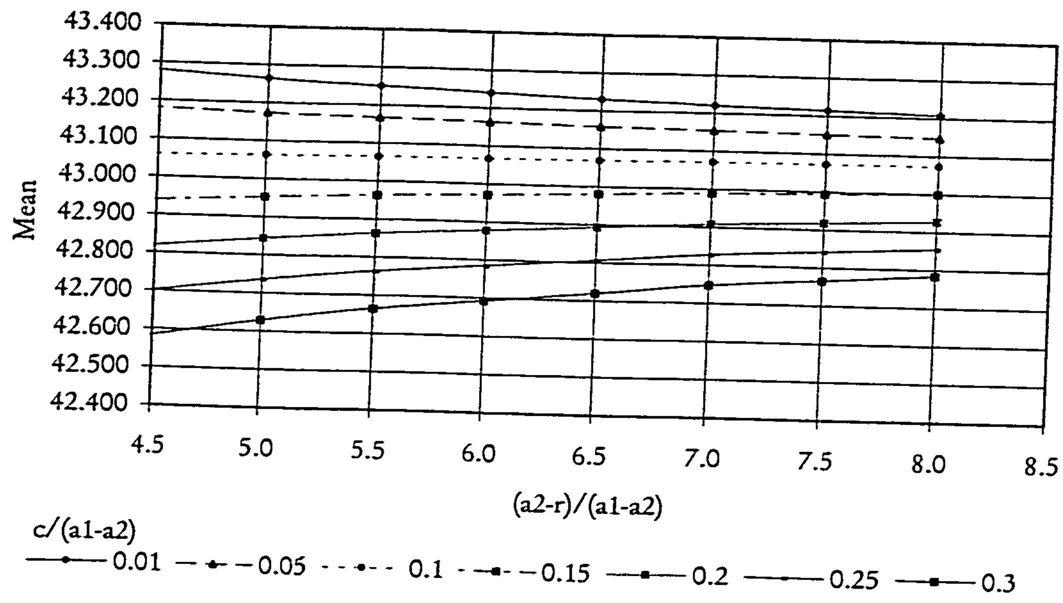


Figure 4-15: Mu versus a2-r/a1-a2 at sigma = 2
Special case I

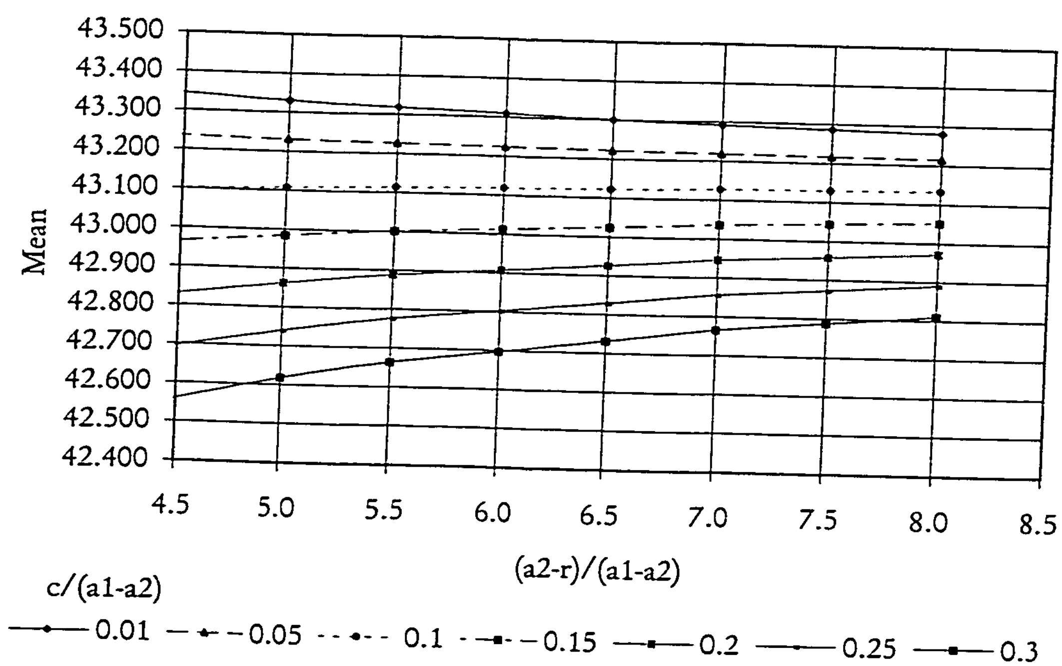


Figure 4-16: Mu versus a2-r/a1-a2 at sigma = 2.25
Special case I

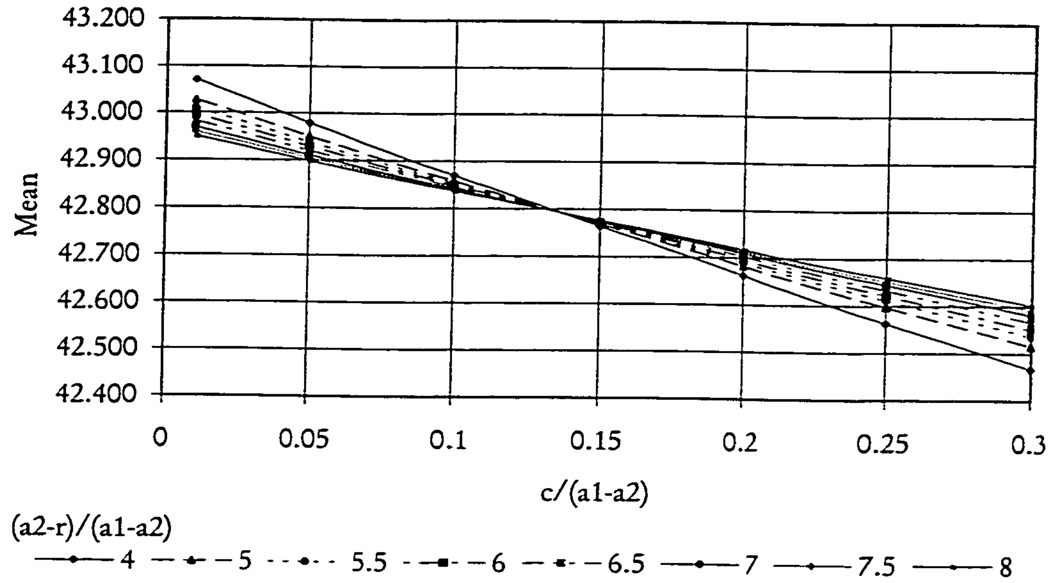


Figure 4-17: Mu versus c/a1-a2 at sigma = 1.5
Special case I

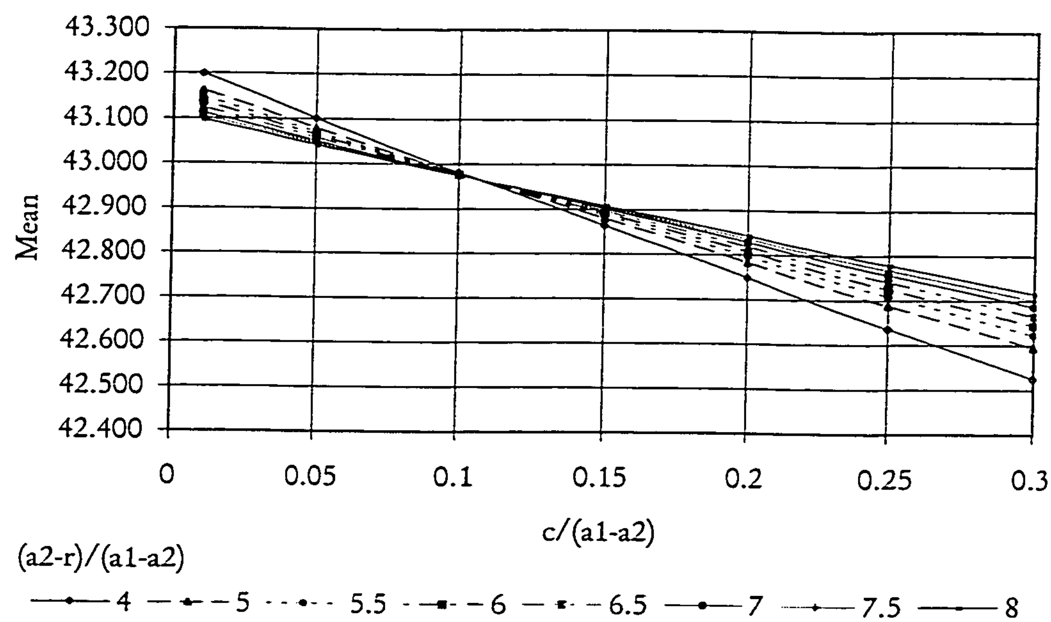


Figure 4-18: Mu versus c/a1-a2 at sigma = 1.75
Special case I

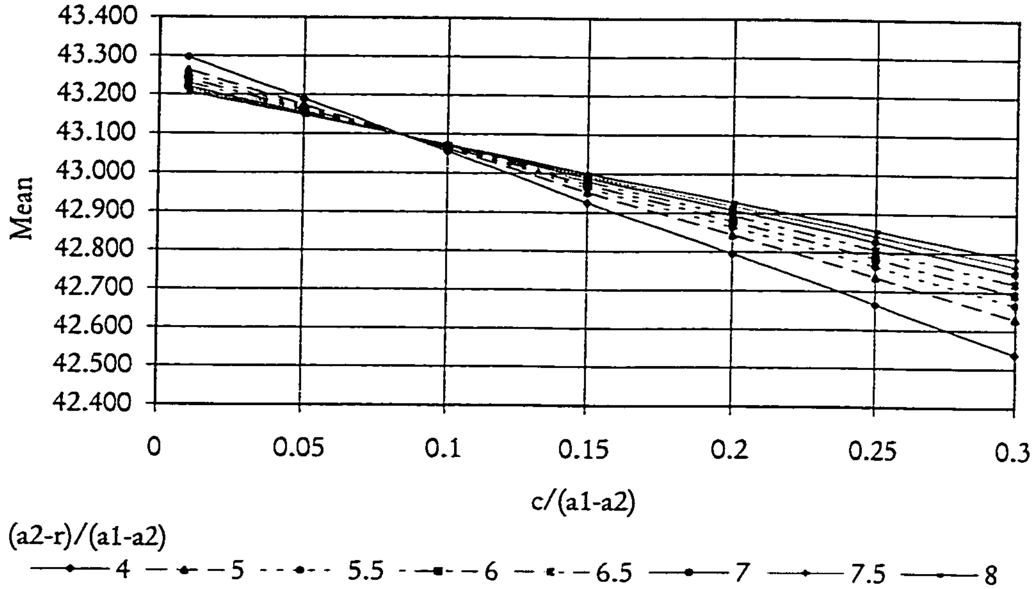


Figure 4-19: Mu versus c/a1-a2 at sigma = 2
Special case I

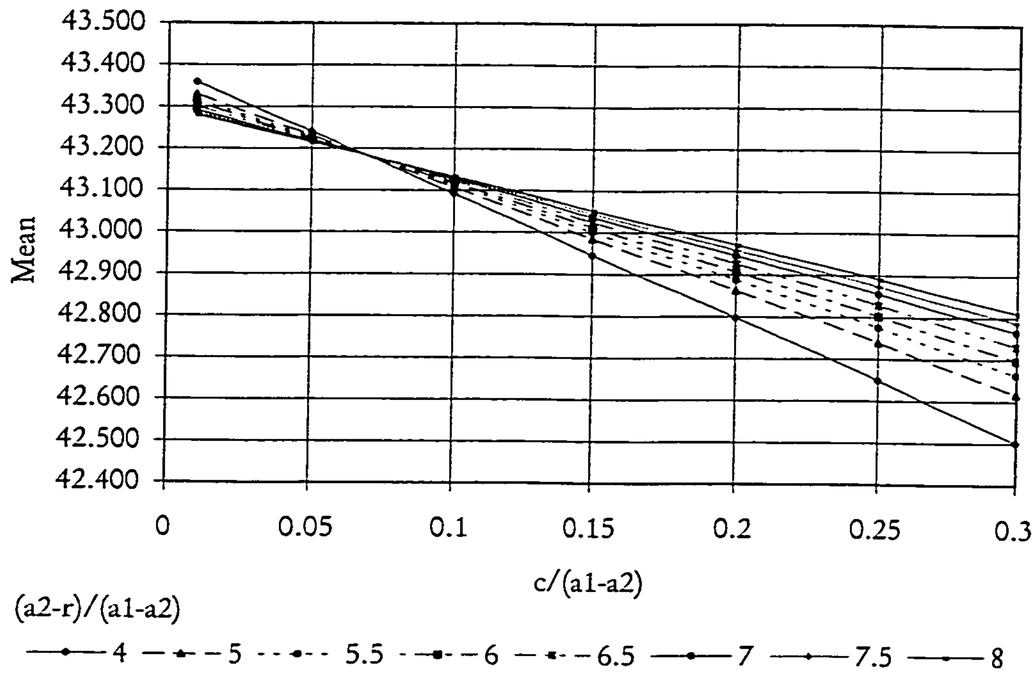


Figure 4-20: Mu versus c/a1-a2 at sigma = 2.25
Special case I

2. rect relationship between the cost of production and the mean of the process.

The effect of $\frac{c}{a_1 - a_2}$ is relatively higher as compared to $\frac{a_2 - r}{a_1 - a_2}$ on expected profit. As discussed in the last section this effect is intuitive as the prices are related to probability of the product falling into that grade while the cost is proportional to the process mean itself.

4.3.2.3 EFFECTS ON EXPECTED PROFIT (SPECIAL CASE II)

The effects on expected profit by varying different parameters are presented in table 4-3. The result shows a higher variation in the expected profit with cost than selling price or σ_y. The results are presented graphically in figures 4-21 to 4-28. In figures 4-21 to 4-24 the expected profit is plotted against $\frac{a_2 - r}{a_1 - a_2}$ the result show a gain in expected profit with increasing $\frac{a_2 - r}{a_1 - a_2}$. This gain is slightly higher with lower costs. Figures 4-25 to 4-28 shows the variation in expected profit with $\frac{c}{a_1 - a_2}$. The result indicates a gain in expected profit with the lowering cost of production. The results are summarized as follows:

1. As the target is set at the specification limit L_1 the mean is forced closer to the target, resulting in a decrease in expected profit as compared to special case I.

TABLE 4-3: Expected profit at different parameter settings

(Special case II)

σ_y	$\frac{a_2 - r}{a_1 - a_2}$	$\frac{c}{a_1 - a_2}$ = 0.01	0.05	0.1	0.15	0.2	0.25	0.3
	4.0	4.7330	3.7004	2.4127	1.1282	-0.1532	-1.4316	-2.7070
	5.0	4.7890	3.9292	2.8566	1.7863	0.7182	-0.3478	-1.4117
	5.5	4.8116	4.0209	3.0343	2.0496	1.0668	0.0858	-0.8933
1.5	6.0	4.8285	4.0896	3.1675	2.2470	1.3283	0.4111	-0.5045
	6.5	4.8455	4.1583	3.3007	2.4445	1.5897	0.7364	-0.1156
	7.0	4.8626	4.2271	3.4339	2.6419	1.8512	1.0617	0.2734
	7.5	4.8740	4.2730	3.5227	2.7736	2.0256	1.2786	0.5327
	8.0	4.8854	4.3188	3.6115	2.9052	2.1999	1.4955	0.7921
	4.0	4.6315	3.5959	2.3047	1.0171	-0.2671	-1.5479	-2.8253
	5.0	4.6881	3.8257	2.7500	1.6768	0.6060	-0.4624	-1.5284
	5.5	4.7108	3.9176	2.9281	1.9407	0.9553	-0.0280	-1.0093
1.75	6.0	4.7279	3.9866	3.0617	2.1386	1.2173	0.2979	-0.6198
	6.5	4.7450	4.0556	3.1952	2.3365	1.4794	0.6238	-0.2302
	7.0	4.7622	4.1246	3.3288	2.5345	1.7415	0.9498	0.1595
	7.5	4.7736	4.1706	3.4179	2.6664	1.9162	1.1672	0.4193
	8.0	4.7851	4.2166	3.5069	2.7984	2.0910	1.3846	0.6793
	4.0	4.5239	3.4861	2.1924	0.9027	-0.3831	-1.6649	-2.9430
	5.0	4.5808	3.7164	2.6385	1.5633	0.4909	-0.5789	-1.6459
	5.5	4.6036	3.8086	2.8169	1.8276	0.8406	-0.1441	-1.1265
2	6.0	4.6208	3.8777	2.9507	2.0258	1.1029	0.1821	-0.7368
	6.5	4.6379	3.9468	3.0846	2.2241	1.3654	0.5084	-0.3469
	7.0	4.6551	4.0159	3.2184	2.4223	1.6278	0.8348	0.0432
	7.5	4.6666	4.0620	3.3076	2.5545	1.8028	1.0524	0.3034
	8.0	4.6781	4.1081	3.3968	2.6867	1.9778	1.2701	0.5636
	4.0	4.4123	3.3731	2.0782	0.7876	-0.4985	-1.7802	-3.0575
	5.0	4.4694	3.6038	2.5246	1.4485	0.3754	-0.6946	-1.7616
	5.5	4.4923	3.6961	2.7032	1.7129	0.7252	-0.2599	-1.2425
2.25	6.0	4.5095	3.7653	2.8371	1.9113	0.9876	0.0663	-0.8529
	6.5	4.5267	3.8345	2.9711	2.1096	1.2501	0.3926	-0.4630
	7.0	4.5439	3.9037	3.1051	2.3080	1.5127	0.7191	-0.0729
	7.5	4.5554	3.9499	3.1944	2.4403	1.6878	0.9368	0.1873
	8.0	4.5669	3.9960	3.2837	2.5726	1.8630	1.1546	0.4476

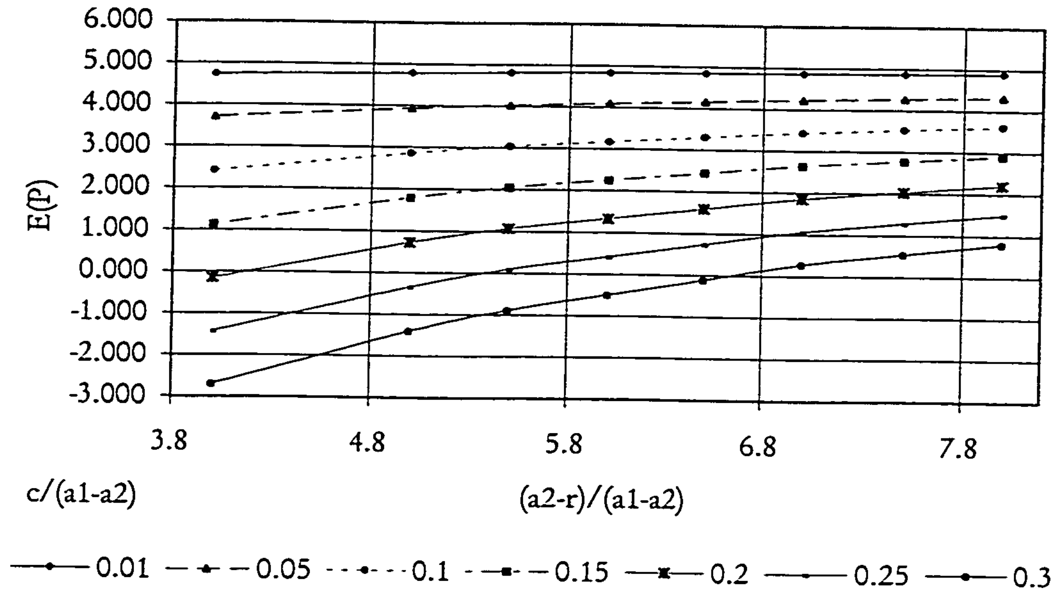


Figure 4-21: E(p) versus a2-r/a1-a2 at sigma = 1.5
Special case II

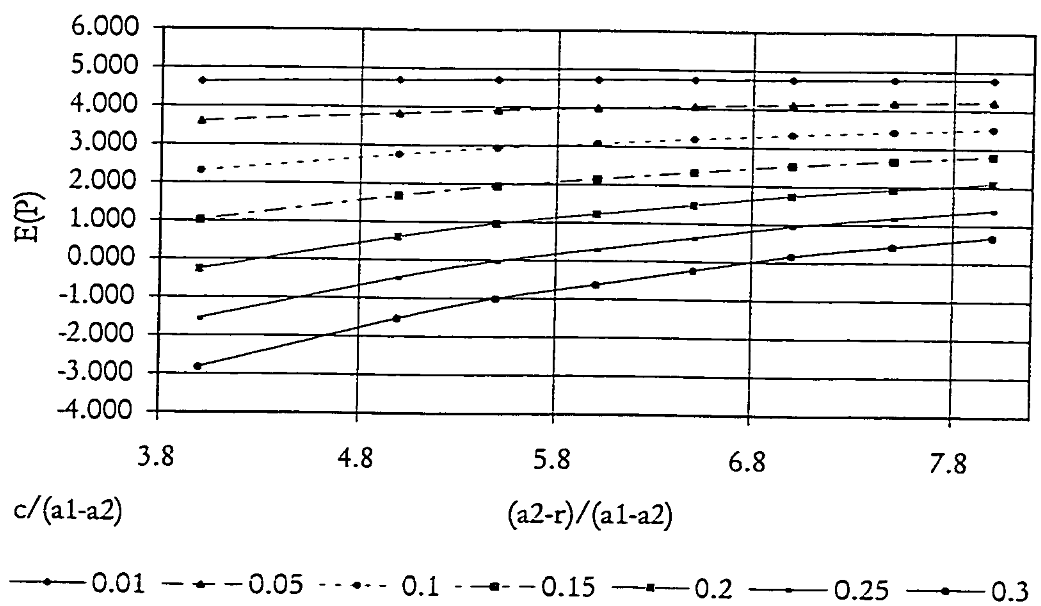


Figure 4-22: E(p) versus a2-r/a1-a2 at sigma = 1.75
Special case II

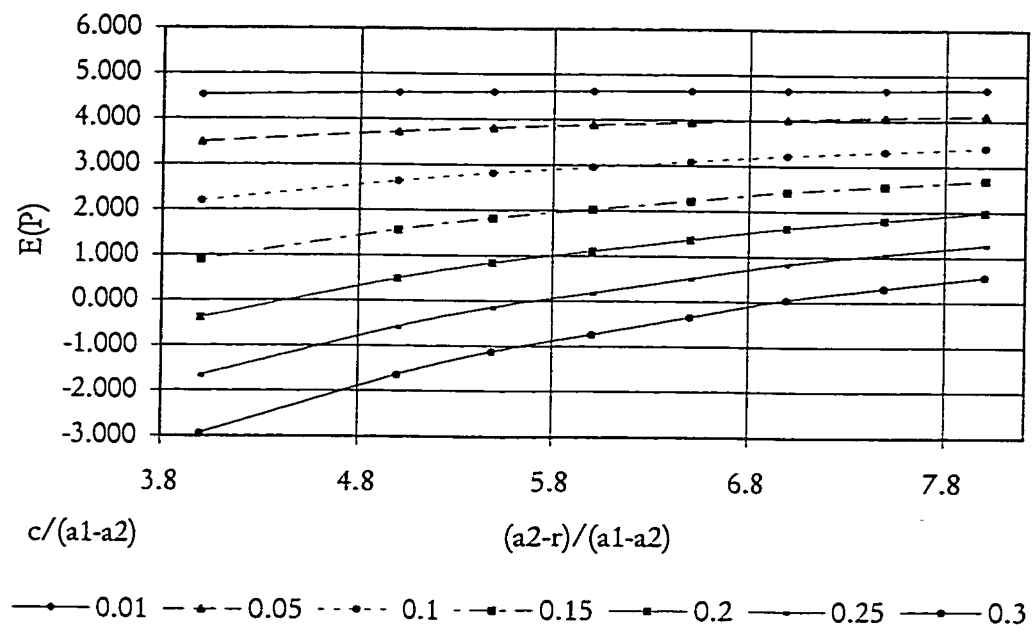


Figure 4-23: E(p) versus a2-r/a1-a2 at sigma = 2
Special case II

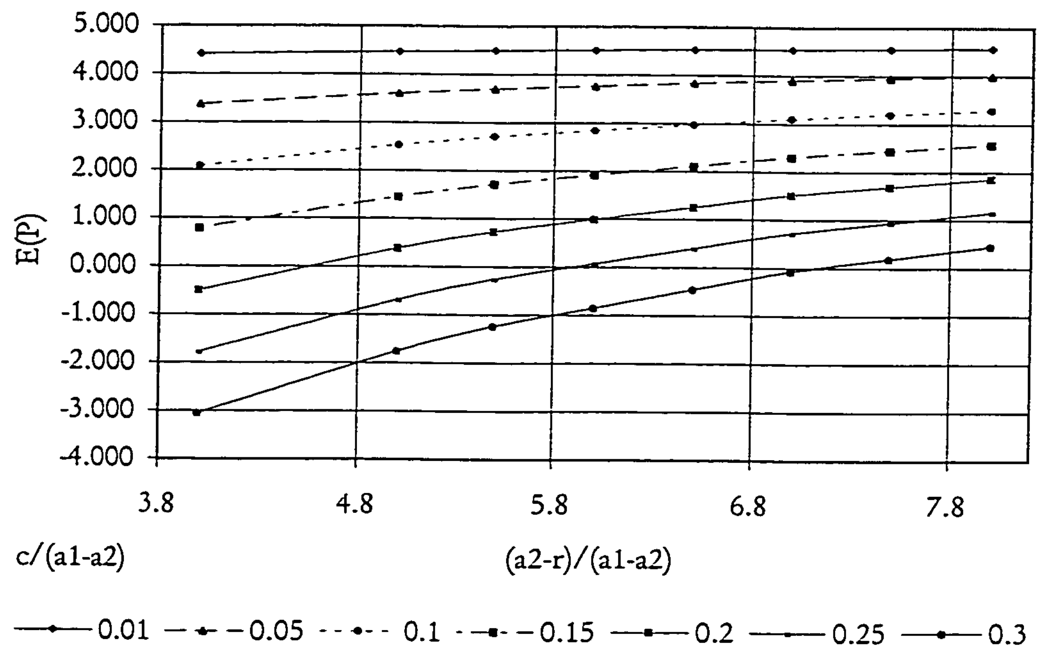


Figure 4-24: E(p) versus a2-r/a1-a2 at sigma = 2.25
Special case II

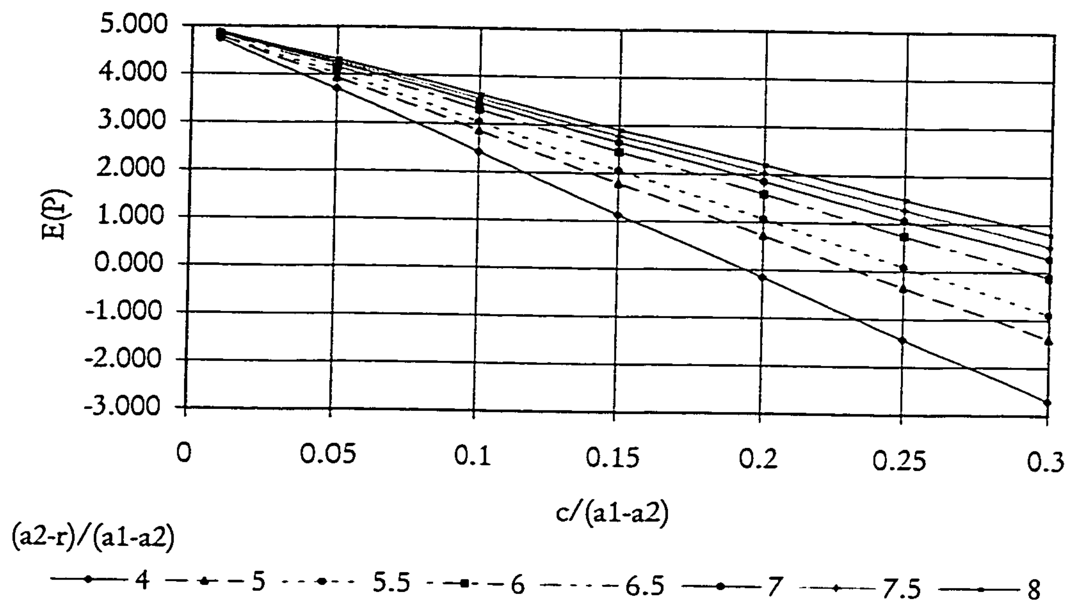


Figure 4-25: E(p) versus c/a1-a2 at sigma = 1.5
Special case II

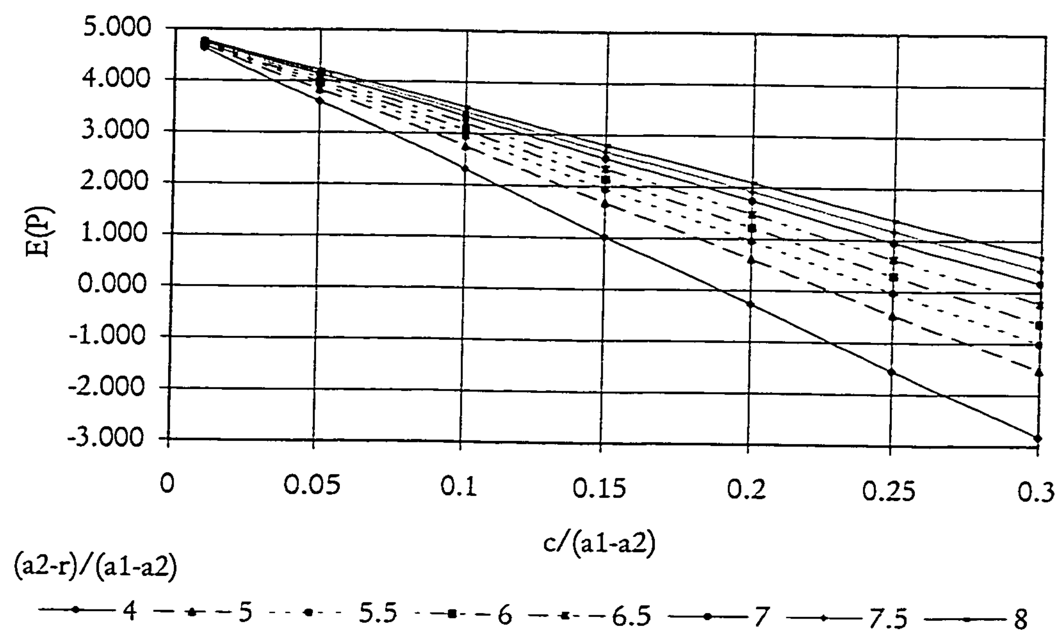


Figure 4-26: E(p) versus c/a1-a2 at sigma = 1.75
Special case II

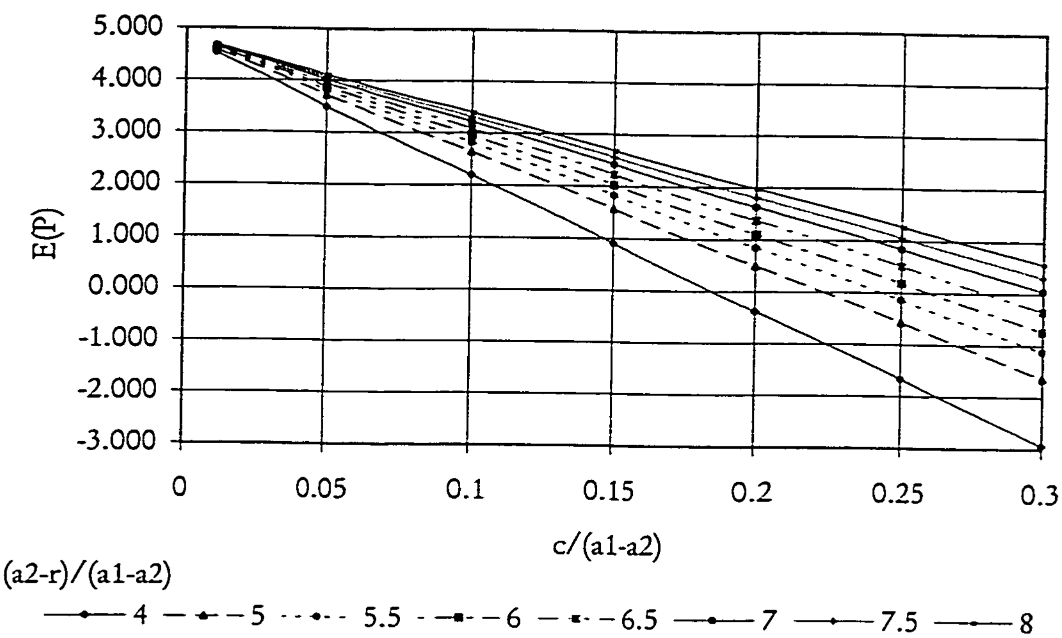


Figure 4-27: E(p) versus c/a1-a2 at sigma = 2
Special case II

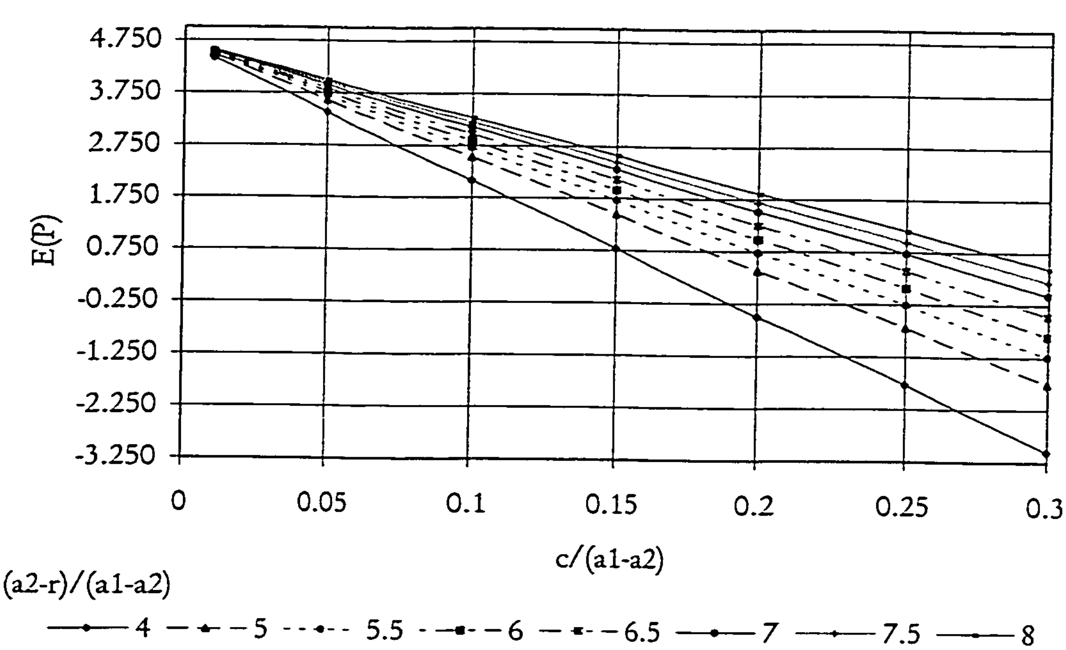


Figure 4-28: E(p) versus c/a1-a2 at sigma = 2.25
Special case II

2. As, in special case one, the expected profit decreases sharply with the increase in $\frac{c}{a_1 - a_2}$. This is again because of the term (-$c\mu$) showing a proportional increase in the production cost.

3. The effect of increasing $\frac{a_2 - r}{a_1 - a_2}$ shows an expected gain in profit. This is because, as the selling price increases gain in profit is intuitive. Also, by setting the price of selling grades higher than scrap, mean pushing the mean higher resulting in more high grade products.

4. The effect of $\frac{c}{a_1 - a_2}$ is relatively higher as compared to $\frac{a_2 - r}{a_1 - a_2}$. The reason behind it evident from the model. The rate of gain in profit is in terms of probability of the product falling in that grade will the cost is directly related to the process mean. Due to this, the effect of change in $\frac{c}{a_1 - a_2}$ is higher than $\frac{a_2 - r}{a_1 - a_2}$. This effect will decrease as the units of measurement becomes proportional to the probabilities i.e., the unit of measurement < 1.

4.3.2.4 EFFECTS ON OPTIMAL MEAN (SPECIAL CASE II)

The effects on optimal mean by varying different parameters are presented in table 4-4. The result shows that the effect of cost is higher on the optimal mean than the selling price. The results are shown graphically in figures 4-29 to 4-36. In figures 4-29 to 4-32 the optimal is plotted against $\frac{a_2 - r}{a_1 - a_1}$ the result show an increase in

TABLE 4-4: Optimal Mean at different parameter settings

(Special case II)

σ_y	$\frac{a_2 - r}{a_1 - a_2}$	$\frac{c}{a_1 - a_2}$ = 0.01	0.05	0.1	0.15	0.2	0.25	0.3
	4.0	43.0701	42.9792	42.8697	42.7642	42.6620	42.5627	42.4658
	5.0	43.0266	42.9503	42.8579	42.7685	42.6818	42.5975	42.5152
	5.5	43.0089	42.9384	42.8530	42.7703	42.6899	42.6117	42.5353
1.5	6.0	42.9955	42.9294	42.8493	42.7716	42.6960	42.6224	42.5506
	6.5	42.9819	42.9204	42.8456	42.7729	42.7023	42.6333	42.5660
	7.0	42.9683	42.9112	42.8418	42.7743	42.7085	42.6443	42.5815
	7.5	42.9591	42.9051	42.8393	42.7752	42.7127	42.6517	42.5920
	8.0	42.9499	42.8989	42.8368	42.7762	42.7170	42.6592	42.6026
	4.0	43.1997	43.1010	42.9807	42.8634	42.7486	42.6358	42.5245
	5.0	43.1621	43.0794	42.9783	42.8796	42.7829	42.6879	42.5943
	5.5	43.1469	43.0706	42.9774	42.8861	42.7967	42.7089	42.6224
1.75	6.0	43.1354	43.0640	42.9766	42.8911	42.8072	42.7248	42.6436
	6.5	43.1239	43.0574	42.9759	42.8961	42.8177	42.7407	42.6648
	7.0	43.1123	43.0507	42.9752	42.9011	42.8283	42.7568	42.6862
	7.5	43.1045	43.0462	42.9747	42.9044	42.8354	42.7675	42.7006
	8.0	43.0968	43.0418	42.9742	42.9078	42.8426	42.7783	42.7150
	4.0	43.2966	43.1883	43.0553	42.9241	42.7944	42.6655	42.5368
	5.0	43.2637	43.1732	43.0617	42.9519	42.8433	42.7356	42.6286
	5.5	43.2504	43.1671	43.0644	42.9631	42.8629	42.7637	42.6651
2	6.0	43.2404	43.1625	43.0663	42.9715	42.8777	42.7848	42.6925
	6.5	43.2304	43.1579	43.0683	42.9799	42.8925	42.8059	42.7200
	7.0	43.2204	43.1532	43.0703	42.9884	42.9073	42.8271	42.7474
	7.5	43.2137	43.1501	43.0716	42.9940	42.9172	42.8412	42.7657
	8.0	43.2070	43.1470	43.0729	42.9997	42.9272	42.8554	42.7841
	4.0	43.3587	43.2396	43.0918	42.9447	42.7975	42.6497	42.5003
	5.0	43.3298	43.2304	43.1070	42.9843	42.8620	42.7396	42.6167
	5.5	43.3182	43.2267	43.1131	43.0002	42.8877	42.7752	42.6626
2.25	6.0	43.3095	43.2239	43.1177	43.0121	42.9069	42.8019	42.6968
	6.5	43.3008	43.2211	43.1222	43.0240	42.9261	42.8285	42.7309
	7.0	43.2920	43.2183	43.1268	43.0359	42.9453	42.8550	42.7648
	7.5	43.2862	43.2165	43.1299	43.0438	42.9581	42.8727	42.7874
	8.0	43.2804	43.2146	43.1329	43.0517	42.9709	42.8904	42.8100

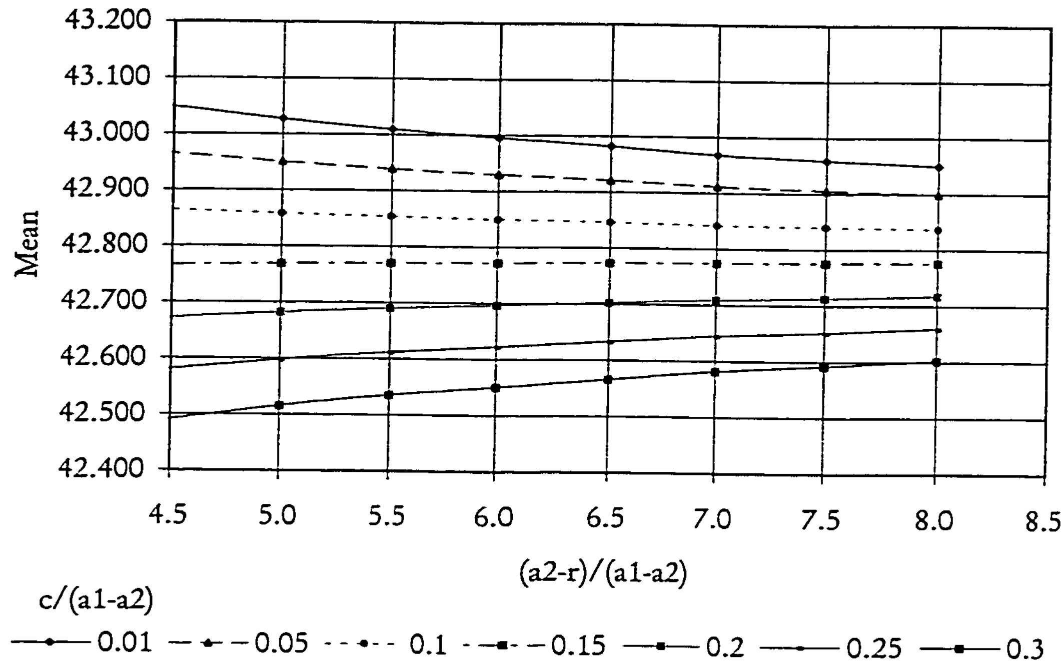


Figure 4-29: Optimal Mean versus a2-r/a1-a2 at sigma = 1.5
Special case II

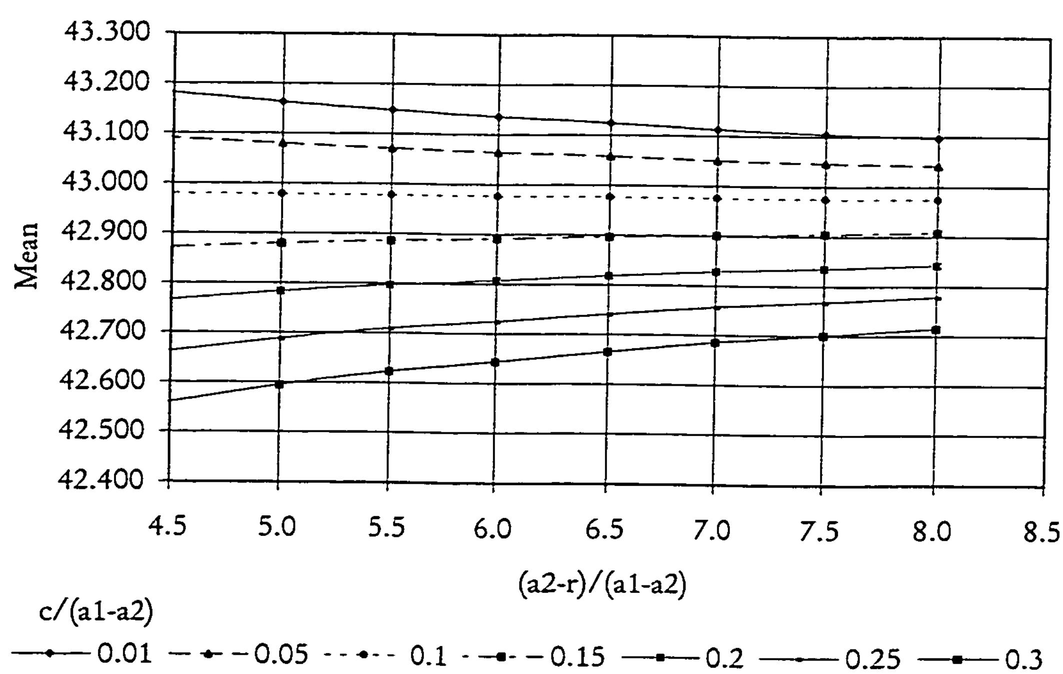


Figure 4-30: Optimal Mean versus a2-r/a1-a2 at sigma = 1.75
Special case II

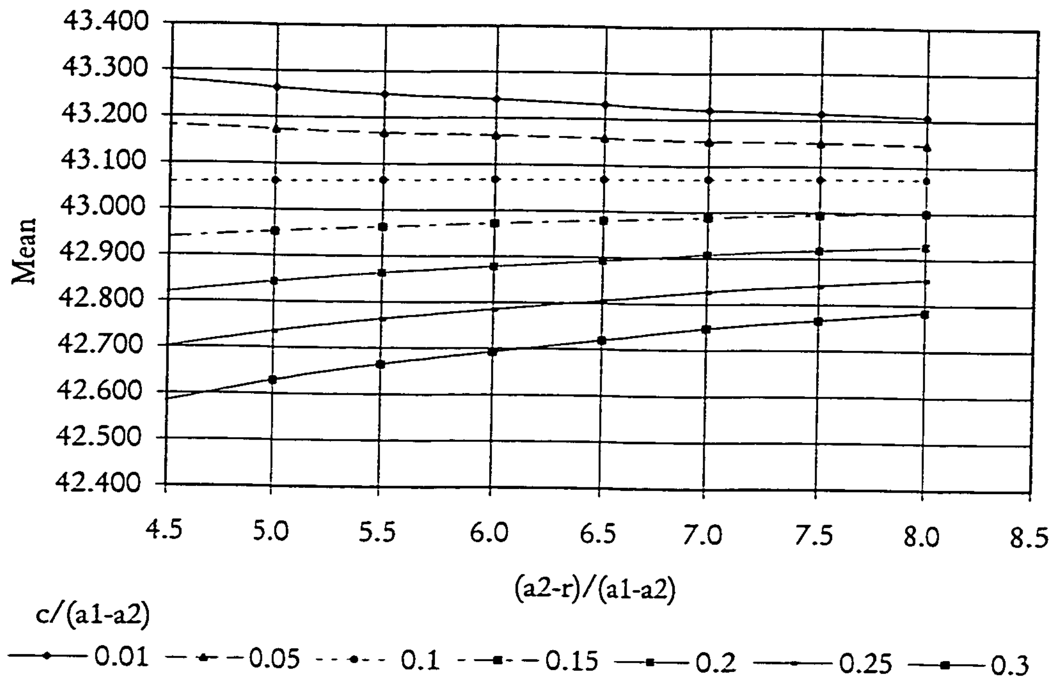


Figure 4-31: Optimal Mean versus a2-r/a1-a2 at sigma = 2
Special case II

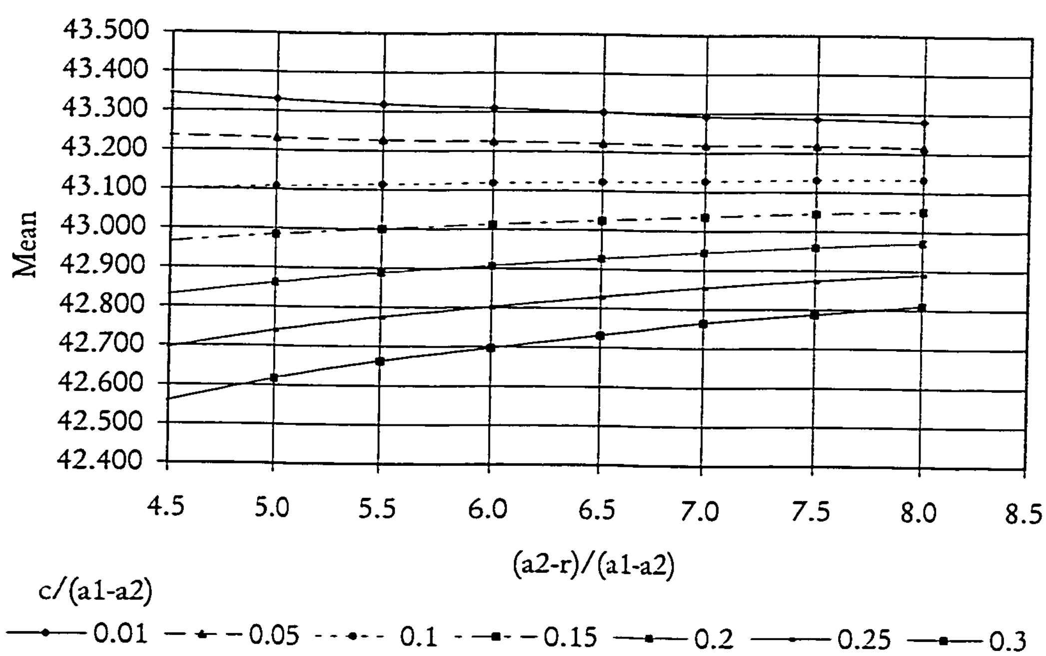


Figure 4-32: Optimal Mean versus a2-r/a1-a2 at sigma = 2.25
Special case II

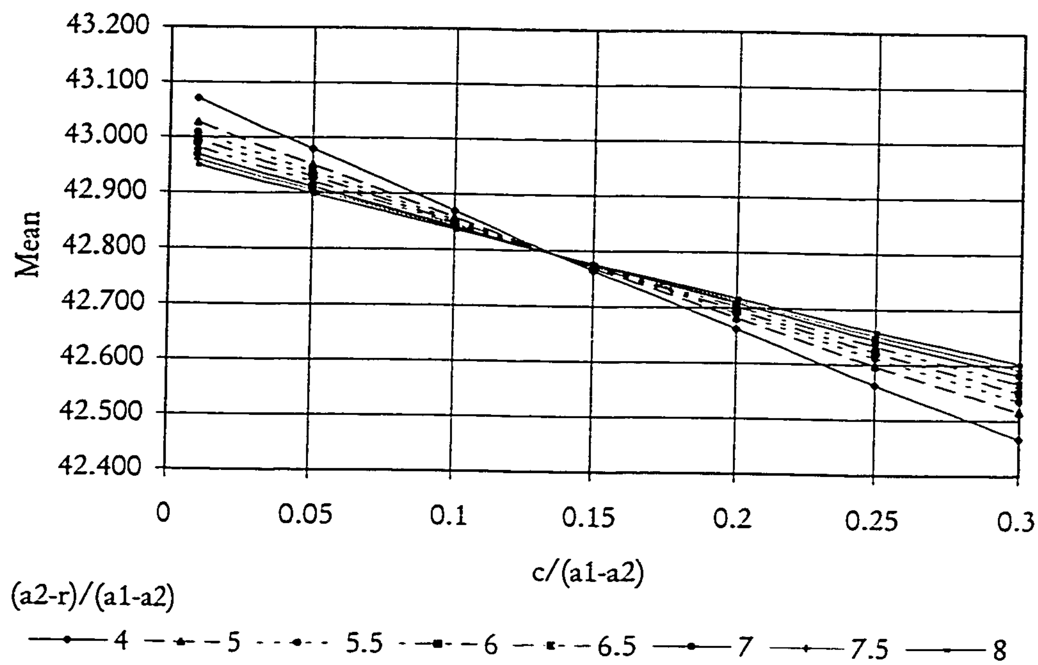


Figure 4-33: Optimal Mean versus c/a1-a2 at sigma = 1.5
Special case II

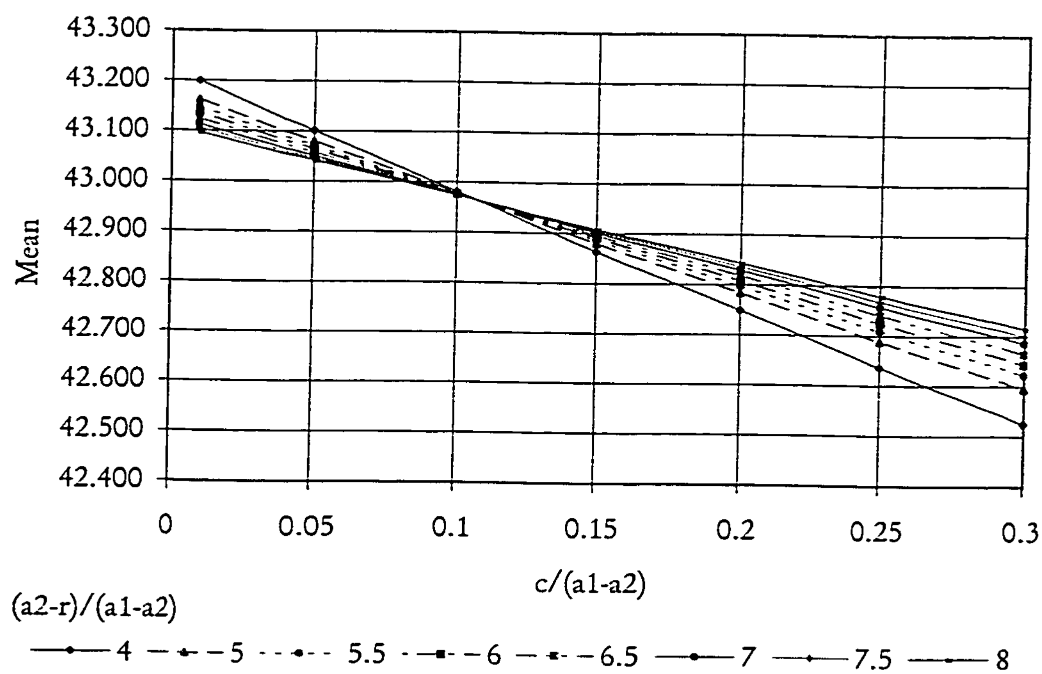


Figure 4-34: Optimal Mean versus c/a1-a2 at sigma = 1.75
Special case II

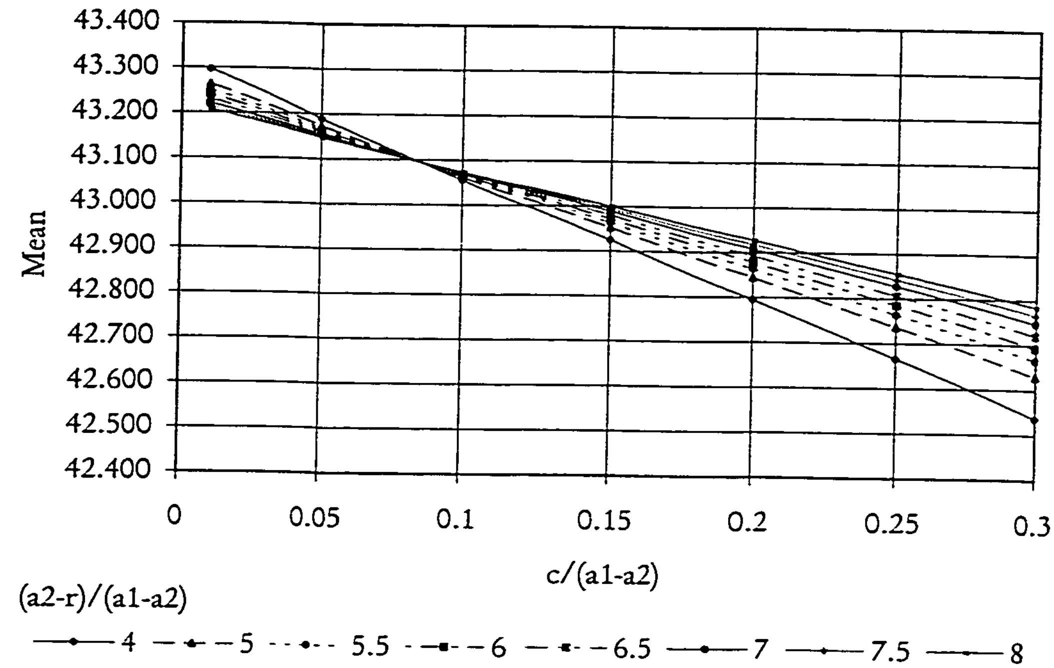


Figure 4-35: Optimal Mean versus c/a1-a2 at sigma = 2
Special case II

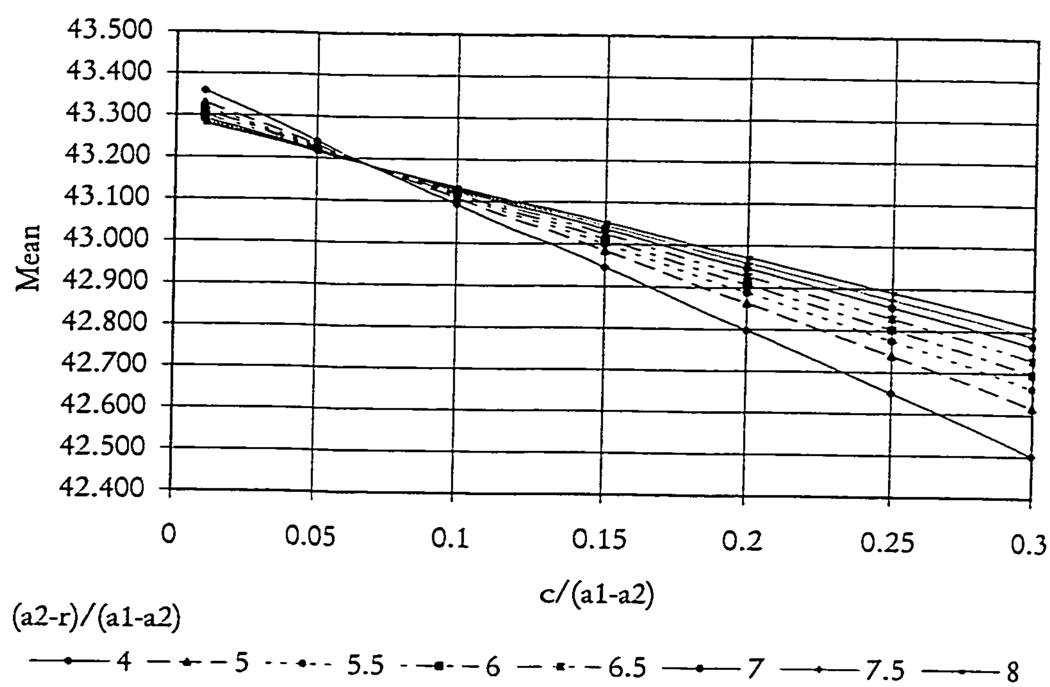


Figure 4-36: Optimal Mean versus c/a1-a2 at sigma = 2.25
Special case II

optimal mean with the increase in $\frac{a_2 - r}{a_1 - a_1}$. In figures, 4-33 to 4-36 the change in optimal mean with the change in cost is plotted. The results are summarized below:

1. As compared to case I, the effect on mean by setting the target at L_1 is quite significant. The mean in this case is forced to set closer to the target L_1 while the effect on optimal mean in case I is minimal. This is because, the target is related to the mean in case I, while in case II the mean is related to the target. In addition, for case I, as the product falling in scrap region is minimal, the effect on the optimal mean is also minimal.

2. The optimal mean increases with the increase in $\frac{a_2 - r}{a_1 - a_2}$. This is because as the selling price of grade increases (keeping the other fixed) consequently the optimal mean will be set higher.

3. The change is optimal mean with cost i.e., $\frac{c}{a_1 - a_2}$ is quite significant as compared to $\frac{a_2 - r}{a_1 - a_2}$. There is sharp decrease in mean with the increase in mean. This is an expected result due to the direct relationship between the cost of production and the mean of the process.

4. The effect of $\frac{c}{a_1 - a_2}$ is relatively higher as compared to $\frac{a_2 - r}{a_1 - a_2}$ on expected profit. As discussed in the last section this effect is intuitive as the prices are related to probability of the product falling into that grade while the cost is proportional to the process mean itself.

5. There is a $\frac{c}{a_1 - a_2}$ level in the process at which the mean becomes independent of the selling price. i.e., the mean would be the same irrespective of the $\frac{a_2 - r}{a_1 - a_2}$ e.g., in figure 4-33 at $\frac{c}{a_1 - a_2}$ = 0.1 the mean remains fixed at 42.98 irrespective of the level of $\frac{a_2 - r}{a_1 - a_2}$.

4.4 CONCLUSION

In this chapter, a model for multi-class screening targeting problem is developed incorporating product uniformity via a Taguchi quadratic loss function. Also a sensitivity analysis was performed to study the effect of model parameters. Two special cases for the target are considered. The results from these two special cases showed the effect of the target selection on the optimal mean. This effect is higher in the second case where the target value is not set at the optimal mean itself. The comparison of this model with the Min Koo Lee and Joon Soon Jang (1997) model will be presented in chapter 6.

CHAPTER 5

PROCESS TARGETING WITH UNIFORMITY PENALTY AND MEASUREMENT ERROR

5.1 INTRODUCTION

The purpose of this chapter is to further generalize the multi-class screening targeting model by integrating Model 2 (EPM2) and Model 3 (EPM3). In model 2, the idealistic assumption of error free measurement system is relaxed and in model 3, a uniformity penalty similar to that of Taguchi quadratic loss function was introduced. This generalization in the model will give the leverage to use a model that can take care of both measurement error and the non-uniformity of the product. The effect of both measurement error and inconsistency may be highly significant, as can be seen from earlier results, and a generalized model is required.

As in model 2, we assume that the quality characteristic is measured and the observed value of the quality characteristic 'X' is not the same as the true value 'Y' due to the presence of measurement error, i.e.,

$$X = Y + \varepsilon$$

Where 'ε' represents error in measurement and it is assumed to be unbiased and normally distributed with known variance σ_ε^2 i.e., $\varepsilon \sim N(0, \sigma_\varepsilon^2)$.

For reducing the effect of the error, "cut off points" for the measurement system are considered as in model 2 i.e., instead of the product, being inspected based on the limits of the grades, these cut off points will be used as the criteria of classification.

The objective of this model is to maximize the expected profit and reducing the effect of error, and inconsistency on the expected profit, by finding the optimal cut off points and the process mean. The objective of this model is to maximize the expected profit by finding the optimal process mean and reducing the effect of error, on the expected profit, by finding the optimal cut off points and reducing the effect of no-consistency.

The rest of the chapter is organized as follows: in the section 5.2, the model development will be discussed, followed by two special cases in section 5.2.4. In the first case, the target value is set at the process mean itself. While in the second special case the process target is assumed to be set at the specification limit of the grade 1 i.e., L_1. Finally, in section 5.4 the results will the discussed. In this section, an illustrative example is presented in section 5.4.1 followed by the sensitivity analysis of the two special cases.

5.2 MODEL DEVELOPMENT

In this section, the two newly developed models, in chapter 3 and chapter 4 will be integrated i.e., multi-class targeting model under error and under uniformity penalty as shown in figure 5-1. In section 5.2.1, the model assumptions will be presented, in section 5.2.2, the relationship between the observed values and the actual quality characteristic will be discussed, followed by the model development in section 5.2.3. Finally, the special cases will be discussed in section 5.2.4.

5.2.1 MODEL ASSUMPTIONS

The assumptions of the model are:

1. A single item is to be sold in two different markets with different cost/profit structures.
2. The inspection process is error prone.
3. The measurement 'X' is assumed to be unbiased and distributed normally across the true value.
4. The measuring instrument is assumed to have a known variance.
5. The quality characteristic 'Y' is assumed normally distributed with unknown process mean μ_y and known variance σ_y^2.
6. The inspection is based on 'X'.
7. $a_1 > a_2 > r$.
8. $L_1 > L_2$
9. The production cost per item is $c_0 + cy$

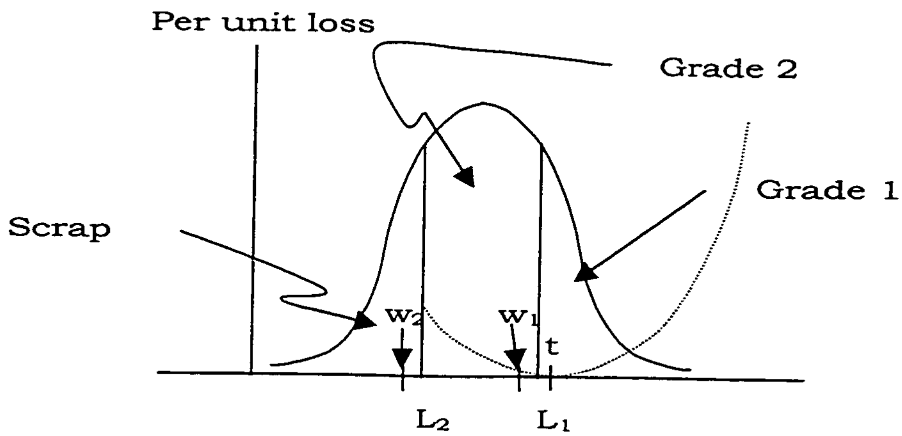


Figure 5-1: A production process with multi class screening & quadratic penalty for uniformity (dotted line). w_i show the cut off values for the inspection.

10. Specification limits on different grade are:

Specification limits on 'Y' for grade 1 are	$Y \geq L_1$,
Specification limits on 'Y' for grade 2 are	$L_2 \leq Y < L_1$,
Specification limits on 'Y' for scrape are	$Y < L_2$,

5.2.2 RELATIONSHIP BETWEEN 'Y' AND 'X'

The relationship between 'X' and 'Y' is already described in section 4-2 i.e., the two random variables are distributed having bivariate normal distribution. The relationship is as follows:

$$h(x, y) = \psi(x, y) = g(x / y; y + \varepsilon, \sigma_x^2) f(y; \mu, \sigma_y^2)$$

$$\psi(y, x) = \frac{1}{2\pi\sigma_y\sigma_\varepsilon\sqrt{1-\rho^2}} e^{-\frac{1}{2\sqrt{1-\rho^2}}\left\{\left(\frac{x-\mu}{\sigma_\varepsilon}\right)^2 + \left(\frac{y-\mu}{\sigma_y}\right)^2 - 2\rho\frac{x-\mu}{\sigma_\varepsilon}\frac{y-\mu}{\sigma_y}\right\}}$$

In standard form:

$$\psi(v, u) = \frac{1}{2\pi} e^{-\frac{1}{2}\left\{(u)^2 + (v)^2 + 2\rho uv\right\}}$$

Where

$$\rho = 1 - \frac{\sigma_\varepsilon^2}{\sigma_x^2} = \frac{\sigma_y^2}{\sigma_x^2} = \frac{\sigma_\varepsilon^2}{\sigma_y^2 + \sigma_\varepsilon^2}$$

5.2.3 STATEMENT OF THE MODEL

Assuming w_i be the cut off value on 'X' therefore if:

$X \geq w_1$	we conclude	$Y \geq L_1$
$w_2 \leq X < w_1$	we conclude	$L_2 \leq Y < L_1$
$X < w_2$	we conclude	$Y < L_2$

The penalties associated in misclassification due to measurement error are:

b_{21}	=	Penalty such that	$X \geq w_1$	while $L_2 \leq Y < L_1$
b_{s1}	=	Penalty such that	$w_2 \leq X < w_1$	while $Y < L_2$
b_{s2}	=	Penalty such that	$X < w_2$	while $Y < L_2$

Now the profit function can be written as:

$$P(y) = \begin{cases} a_1 - (c_0 + cy + c_i) & X \geq w_1 & Y \geq L_1 \\ a_1 - (c_0 + cy + c_i) - b_{21} & X \geq w_1 & L_2 \leq Y < L_1 \\ a_1 - (c_0 + cy + c_i) - b_{s1} & X \geq w_1 & Y < L_2 \\ a_2 - (c_0 + cy + c_i) & w_2 \leq X \leq w_1 & L_2 \leq Y < L_1 \\ a_2 - (c_0 + cy + c_i) & w_2 \leq X \leq w_1 & Y \geq L_1 \\ a_2 - (c_0 + cy + c_i) - b_{s2} & w_2 \leq X \leq w_1 & Y < L_2 \\ r - (c_0 + cy + c_i) & X \leq w_2 & Y < L_2 \\ r - (c_0 + cy + c_i) & X \leq w_2 & Y \geq L_1 \\ r - (c_0 + cy + c_i) & X \leq w_2 & L_2 \leq Y < L_1 \\ -K(y-t)^2 & -\infty \leq X \leq \infty & Y \geq L_2 \end{cases}$$

The profit function shows three possibilities for each grade i.e., for grade 1: the possibilities are 1) to be classified correctly, 2) grade 2 misclassified as grade 1 & 3) scrape misclassified as grade 1. Similarly for grade 2: the possibilities are 1) to be classified correctly, 2) grade 1 misclassified as grade 2 & 3) scrape misclassified as grade 2. For scrape: the possibilities are 1) to be classified correctly, 2) grade 1 misclassified as scrape & 3) grade 2 misclassified as scrape. If a lower grade item is misclassified as higher grade, the loss is represented by the penalties associated with it i.e., b_{ij}, on the other hand if a higher grade item is misclassified as a lower grade the loss is reflected in the loss of profit i.e., selling at lower price. The 'K' is defined as per squared deviation penalty per item. The term $(y - t)^2$ determines the squared deviation in the product. If the quality characteristic 'Y' lies exactly on target 't' this deviation term vanishes there will be no penalty for inconsistency. Therefore, the expected profit per item can be written as:

$$
\begin{aligned}
E\{P(\mu, w_1, w_2, t)\} = & \int_{w_1}^{\infty}\int_{L_1}^{\infty}(a_1 - c_0 - cy - c_i)\varphi(x,y)dydx \\
& + \int_{w_1}^{\infty}\int_{L_2}^{L_1}(a_1 - c_0 - cy - c_i - b_{21})\varphi(x,y)dydx \\
& + \int_{w_1}^{\infty}\int_{-\infty}^{L_2}(a_1 - c_0 - cy - c_i - b_{s1})\varphi(x,y)dydx + \int_{w_2}^{w_1}\int_{L_2}^{L_1}(a_2 - c_0 - cy - c_i)\varphi(x,y)dydx \\
& + \int_{w_2}^{w_1}\int_{L_1}^{\infty}(a_2 - c_0 - cy - c_i)\varphi(x,y)dydx + \int_{w_2}^{w_1}\int_{-\infty}^{L_2}(a_2 - c_0 - cy - c_i - b_{s2})\varphi(x,y)dydx \\
& + \int_{-\infty}^{w_2}\int_{-\infty}^{\infty}(r - c_0 - cy - c_i)\varphi(x,y)dydx - \int_{-\infty}^{\infty}\int_{L_2}^{\infty}K(y-t)^2\varphi(x,y)dydx
\end{aligned}
$$

The first seven terms constitute Model 2 (EPM2). Rewriting the above equation as:

$$E\{P(\mu, w_1, w_2, t)\} = EPM2 - \int_{L_2}^{\infty}\left[K(y-t)^2\right]\varphi\, dy \tag{5-1}$$

Let,

$$(y-t)^2 = [(y-\mu)+(\mu-t)]^2 = (y-\mu)^2 + 2(y-\mu)(\mu-t) + (\mu-t)^2$$

Using above relationship the profit function becomes:

$$\begin{aligned} E\{P(\mu, w_1, w_2, t)\} &= EPM2 \\ &- K\left[\int_{L_2}^{\infty}(y-\mu)^2\varphi\, dy + 2(\mu-t)\int_{L_2}^{\infty}(y-\mu)\varphi\, dy + (\mu-t)^2\int_{L_2}^{\infty}\varphi\, dy\right] \end{aligned} \tag{5-2}$$

Let,

$$z = \frac{y-\mu}{\sigma_y}$$

$$\Rightarrow \qquad y = \mu + z\sigma$$

The equation (5-2) can be restated as:

$$\begin{aligned} E\{P(\mu, w_1, w_2, t)\} &= EPM2 \\ &- K\left[\int_{\Gamma_2}^{\infty}(\sigma z)^2\varphi(z)dz + 2(\mu-t)\int_{\Gamma_2}^{\infty}(\sigma z)\varphi(z)dz + (\mu-t)^2\int_{\Gamma_2}^{\infty}\varphi(z)dz\right] \end{aligned}$$

or

$$\begin{aligned} EPM4 &= EPM2 \\ &- K\left[\varphi(\Gamma_2)\left[\sigma^2 z\varphi(\Gamma_2) + 2\sigma(\mu-t)\right] + \left[\sigma_Y^2 + (\mu-t)^2\right]\left[1-\Phi(\Gamma_2)\right]\right] \end{aligned} \tag{5-3}$$

5.2.4 SPECIAL CASES

In this section, two special cases will be derived from Model 4 (EPM4). In case I, the process target is at the mean of the process while in the case II the target is set at L_1. These special cases are as follows:

5.2.4.1 SPECIAL CASE I {MODEL 4-I ($EPM4_I$)}

In this case, the target is assumed to be at the mean of the process i.e. $t=\mu$. (See figure 5-2(a))

Letting $E\{P(\mu, w_1, w_2)\} \equiv EPM4_I$ we can rewrite the equation (5-3) as:

$$EPM4 = EPM2 \\ -K\left[\varphi(\Gamma_2)\left[\sigma^2 z\varphi(\Gamma_2)+2\sigma(\mu-\mu)\right]+\left[\sigma_Y^2+(\mu-\mu)^2\right]\left[1-\Phi(\Gamma_2)\right]\right]$$

or

$$EPM4 = EPM2 \\ -K\left[\varphi(\Gamma_2)\left[\sigma^2 z\varphi(\Gamma_2)\right]+\sigma_Y^2\left[1-\Phi(\Gamma_2)\right]\right]$$

5.2.4.2 SPECIAL CASE II {MODEL 4-II ($EPM4_{II}$)}

In this case, the target for uniformity i.e., 't' is assumed to be set at the specification limit of the grade 1 i.e., $t=L_1$. (See figure 5-2(b))

Letting $E\{P(\mu, w_1, w_2)\} \equiv EPM4_{II}$ we can rewrite the equation (5-3) as:

$$EPM4 = EPM2 \\ -K\left[\varphi(\Gamma_2)\left[\sigma^2 z\varphi(\Gamma_2)+2\sigma(\mu-L_1)\right]+\left[\sigma_Y^2+(\mu-L_1)^2\right]\left[1-\Phi(\Gamma_2)\right]\right]$$

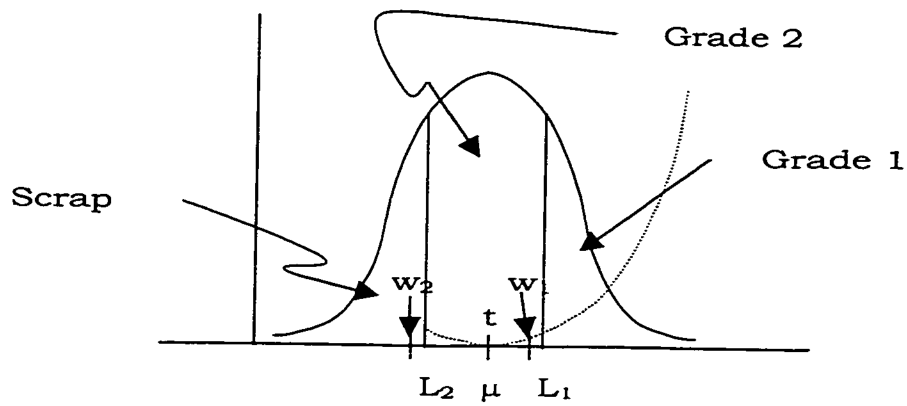


Figure 5-2(a): Special case I ($EPM4_I$) (target at μ)

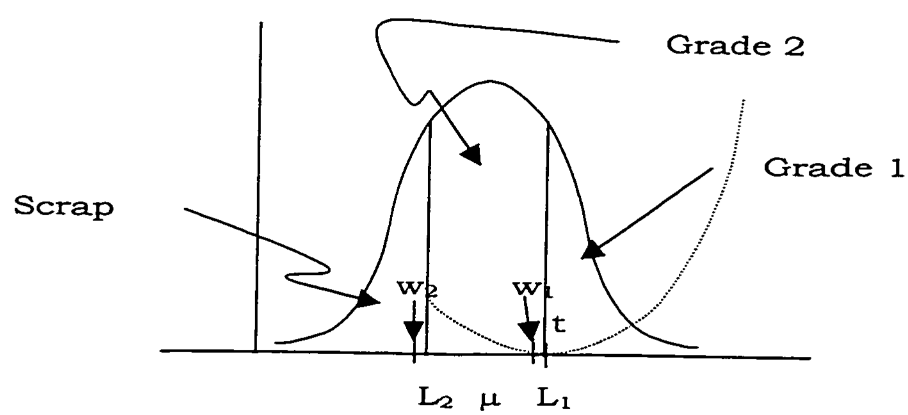


Figure 5-2(b): Special case II ($EPM4_{II}$) (target as L_1)

5.3 RESULTS AND ANALYSIS

In this section, an illustrative example will be presented. This will be followed by the sensitivity analysis. For the numerical analysis, 'FindMinimum' function of Mathematica 4.0 is used. The function uses modified Powell's method. The analysis is performed on a Compaq Deskpro, Pentium III computer with 64 MB of RAM. See appendix B for the notebook (program code).

5.3.1 ILLUSTRATIVE EXAMPLE

Consider a packing plant of a cement factory. The plant consists of two processes a filling process and an inspection process. Each cement bag processed by filling machine is moved to the loading and dispatching phase on the conveyor belt. Inspection is performed by automatic weighing system. Suppose that the cost components and the specification limits are a_1=\$5.5, a_2=\$5.07, r=\$2.5, c_0=\$0.1, c_i=\$0.04, c=\$0.043, b_{s1}=\$5, b_{s2}=\$5.5, b_{21}=\$5, L_1=41.5 kg, L_2=40.0 kg, k=0.05 and $\sigma_y^2 = (1.25)^2$. The error in the measuring system is represented by the correlation co-efficient having the value $\rho = 0.85$ i.e., $\sigma_e^2 = (0.525)^2$.

The expected profit and the optimal values of the mean and the cut off values are found out to be:

Model 1:

E(p) = \$5.16543/unit

μ = 44.8421 kg

Case I: target at μ

E(p) = \$5.09767/unit

μ = 45.67775 kg, w_1 = 41.1753 kg

w_2 = 37.7744 kg

Case II: target at L_1

E(p) = \$4.80841/unit

μ = 43.25022 kg

w_1 = 42.1073 kg

w_2 = 38.7098 kg

Consider the case where the penalty for non-uniformity and the effect of error was neglected, the gain in expected profit will be reflected, if the mean obtained from model 1 is substituted in models obtained.

Case I: target at μ

E(p) = \$5.07661/unit

Gain in profit: \$0.023/unit or approximately 0.42 % gain.

Case II: target at L_1

E(p) = \$4.51807/unit

Gain in profit: \$0.30841/unit or approximately 6.5 % gain.

Point to note is that, the value of K is taken very small and, as in model 3, the effect on profit is small if the k is set very small for special case I, on the other hand if target is set at L_1 the gain per unit is more than 5%.

In addition, as can be seen from figure 5-3 (c), the mean is forced very close to the target L_1 in order to have the maximum number of item close to the target L_1.

5.3.2 SENSITIVITY ANALYSIS

In this parametric analysis, the effect of different parameters on the output values i.e., the expected profit, the optimal mean & the

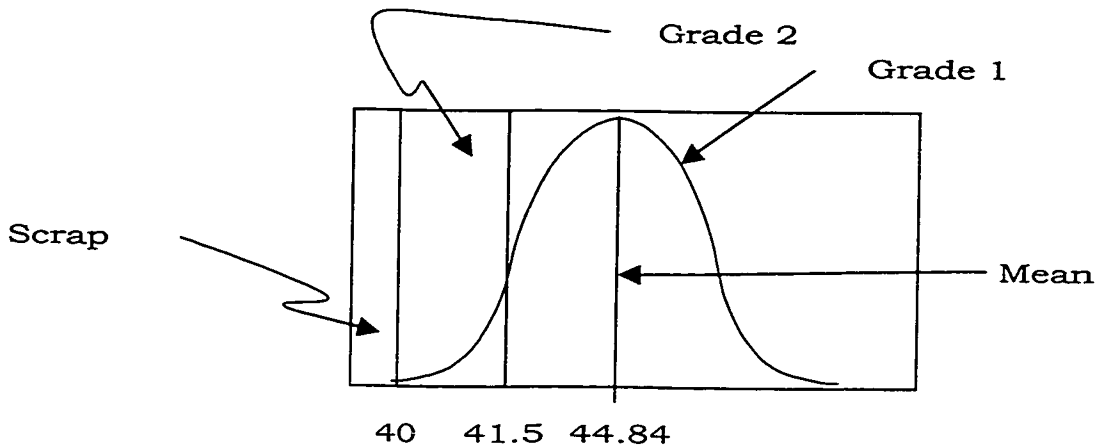


Figure 5-3 (a): Optimal mean settings obtained from model 1

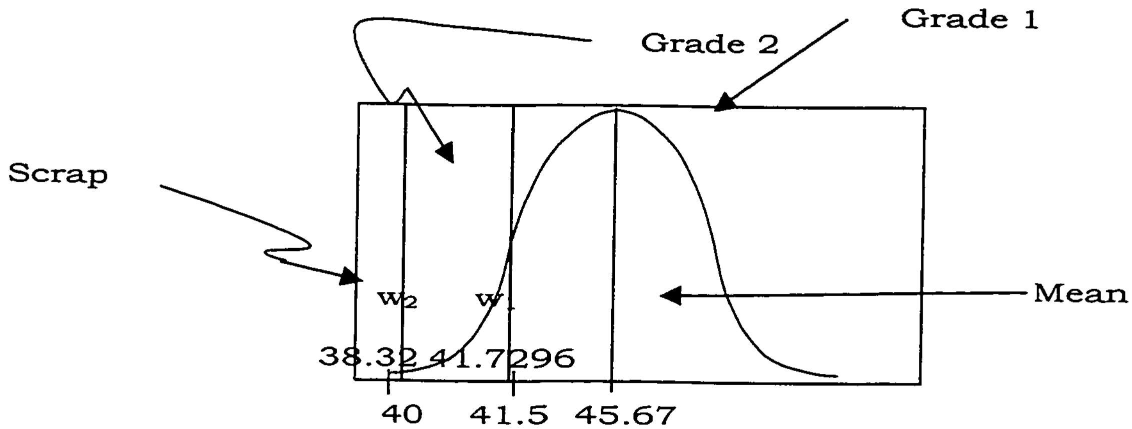


Figure 5-3 (b): Optimal mean obtained from model 4 case I

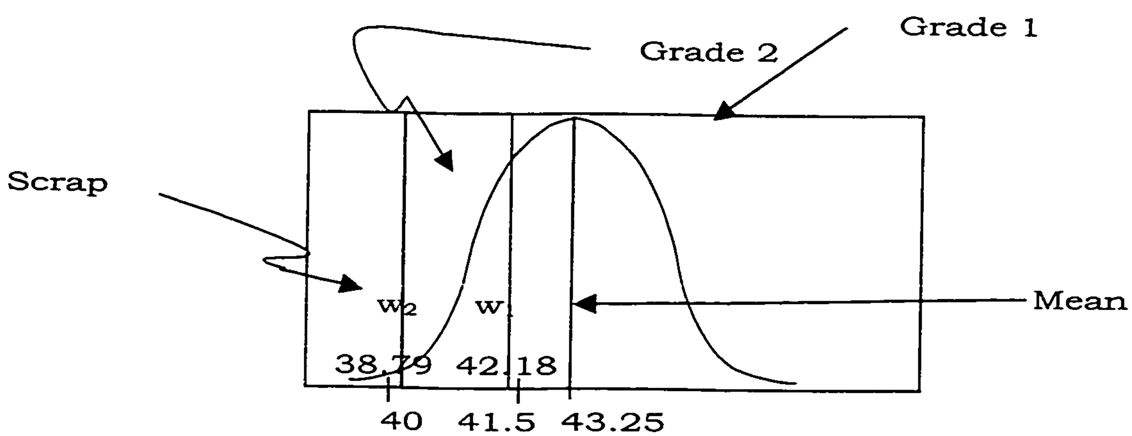


Figure 5-3 (C): Optimal mean obtained from model 4 case II

optimal cut off values are studied. Sensitivity analyses for both the special cases were performed. As there are a number of parameters, a few of them were chosen. These parameters are: $\frac{c}{a_1 - a_2}, \frac{a_2 - r}{a_1 - a_2}, \sigma_y$ and ρ. The main emphasis is on the effect of error i.e., five levels of ρ are taken at 0.85, 0.89, 0.93, 0.97 & 0.995 to cover the effect from low accuracy to high accuracy on the model outputs. Other parameters are taken at two or three levels.

Table 5-1 to Table 5-4 shows the tabulated results for special case I, i.e., the case where the mean is taken as target. While Table 5-5 – 5-8 cover the special case where the L_1 (specification limit of grade 1)

In table 5-1 & 5-4 the expected profit for at different parameter levels are shown for the respective cases. In Table 5-2 to Table 5-5 the optimal mean, for the same cases is presented. The optimal cut off points are shown in Table 5-3 to Table 5-7. The processing time for each problem is also recorded in order to study the computational time of the model. These times are shown in Table 5-4 to Table 5-8 for both the cases respectively.

5.3.2.1 EFFECTS ON EXPECTED PROFIT (SPECIAL CASE I)

In Table 5-1, the expected profit at different parameter levels is shown. The results show that at a given level of $\frac{c}{a_1 - a_2}, \frac{a_2 - r}{a_1 - a_2}$ & σ_y while k is taken at 0.05. As in model 2, the effect of the measurement error is minimized i.e., the level of profit is kept at almost the same level with the change in measurement error. This effect decreases relatively with the increase in per unit production

TABLE 5-1: Expected profit at different parameter settings

(Special case I)

σ_y	$\frac{a_2 - r}{a_1 - a_2}$	ρ	$\frac{c}{a_1 - a_2}$ 0.01	0.15	0.3
1.25	4	0.85	5.00622	1.25918	-2.68161
		0.89	5.00654	1.26709	-2.66400
		0.93	5.00705	1.27791	-2.64114
		0.97	5.00798	1.29473	-2.60733
		0.995	5.00936	1.31630	-2.56564
	6	0.85	5.09767	2.59283	-0.04253
		0.89	5.09792	2.59887	-0.02956
		0.93	5.09833	2.60697	-0.01310
		0.97	5.09905	2.61924	0.01067
		0.995	5.10009	2.63456	0.03937
	8	0.85	5.12974	3.06071	0.88350
		0.89	5.12997	3.06597	0.89463
		0.93	5.13033	3.07298	0.90862
		0.97	5.13096	3.08345	0.92860
		0.995	5.13185	3.09636	0.95248
1.75	4	0.85	4.92162	1.08981	-2.91124
		0.89	4.92207	1.10016	-2.88950
		0.93	4.92279	1.11430	-2.86092
		0.97	4.92411	1.13633	-2.81786
		0.995	4.92606	1.16495	-2.76374
	6	0.85	5.01608	2.45144	-0.22939
		0.89	5.01645	2.45913	-0.21410
		0.93	5.01702	2.46931	-0.19452
		0.97	5.01802	2.48469	-0.16562
		0.995	5.01946	2.50420	-0.12965
	8	0.85	5.04923	2.92964	0.71297
		0.89	5.04956	2.93627	0.72585
		0.93	5.05007	2.94491	0.74217
		0.97	5.05094	2.95779	0.76600
		0.995	5.05218	2.97395	0.79552

cost The results can be shows graphically in figures 5-4 to 5-9. Each figure shows per unit expected profit versus measurement error (represented by ρ) containing three plots at three different levels of cost represented by $\frac{c}{a_1 - a_2}$. Per unit, expected profit decreases sharply with cost as can be seen from the figures 5-4 to 5-9. In each figure the level of $\frac{a_2 - r}{a_1 - a_2}$ and σ_y is fixed. The variation in per unit expected profit with $\frac{a_2 - r}{a_1 - a_2}$ at given level of σ_y (two level), as can be seen by comparing figures 5-4 to 5-6 and figures 5-7 to 5-9, is less as compared to $\frac{c}{a_1 - a_2}$.

The results can be summarized as follows:

1. The model performed very well in nullifying the effect of error in measurement (at any given level of $\frac{c}{a_1 - a_2}, \frac{a_2 - r}{a_1 - a_2}$ & σ_y).

2. $\frac{c}{a_1 - a_2}$, significantly reduces per unit expected profit as expected (cost of production is proportional to the level of the process mean).

3. The effect of increase in $\frac{a_2 - r}{a_1 - a_2}$ is less on per expected profit, as compared to $\frac{c}{a_1 - a_2}$. This is because the cost is directly related to mean while the selling prices are proportional to the probability of the product falling in the respective grade.

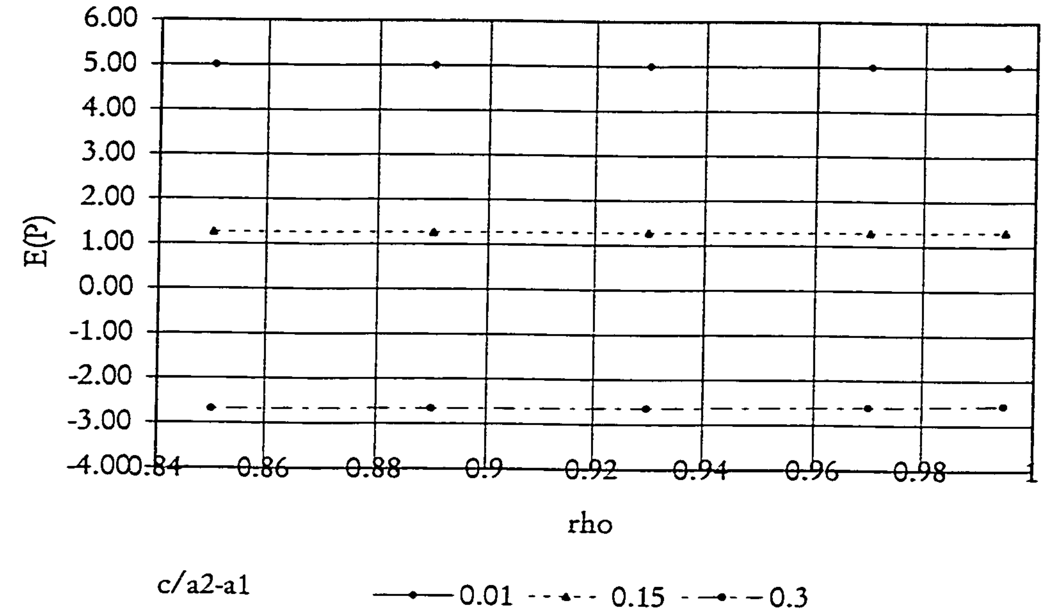


Figure 5-4: E(p) versus measurment error (rho) at a2-r/a1-a2 = 4
sigma = 1.25
Special case I

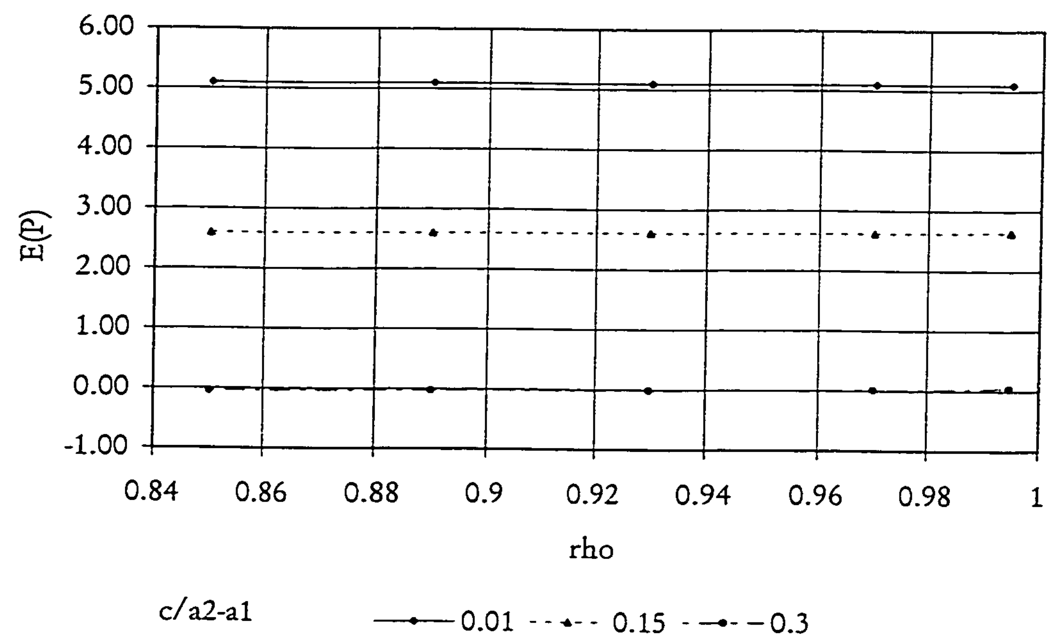


Figure 5-5: E(p) versus measurment error (rho) at a2-r/a1-a2 = 6.5
sigma = 1.25
Special case I

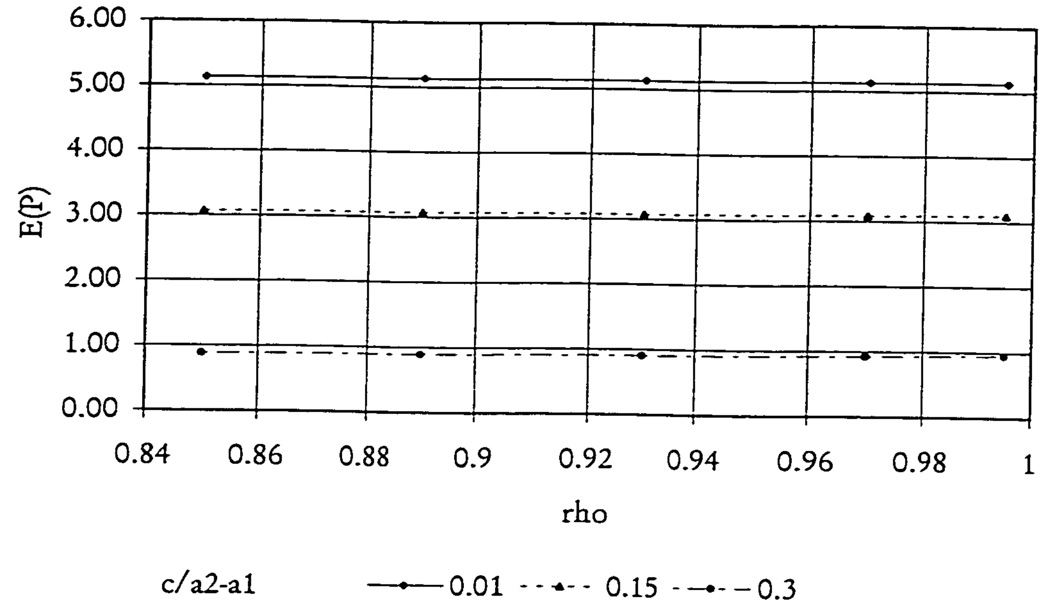


Figure 5-6: E(p) versus measurment error (rho) at a2-r/a1-a2 = 8
sigma = 1.25
Special case I

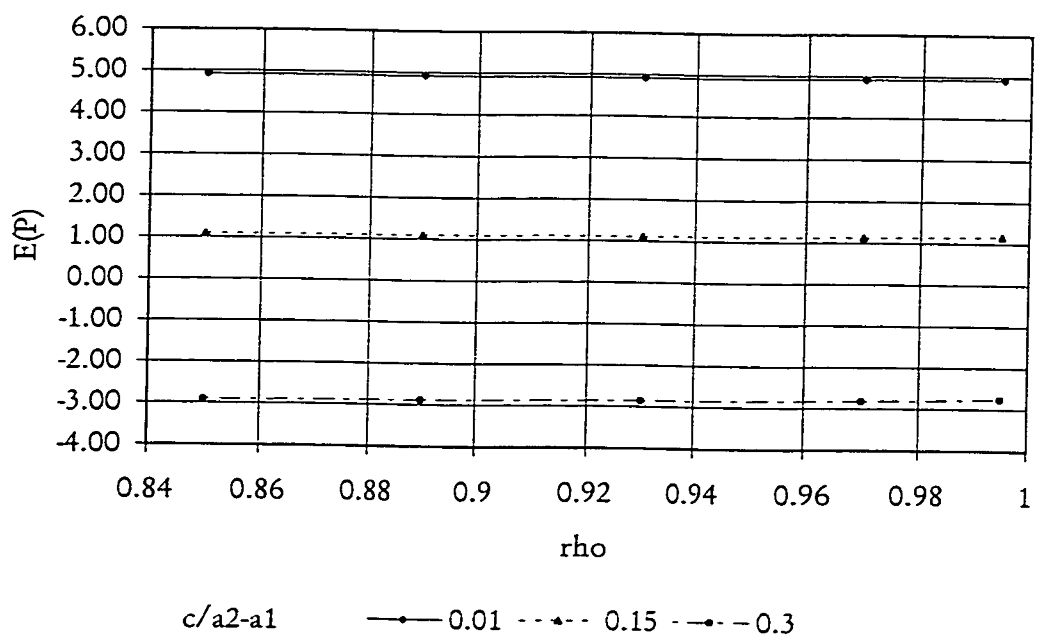


Figure 5-7: E(p) versus measurment error (rho) at a2-r/a1-a2 = 4
sigma = 1.75
Special case I

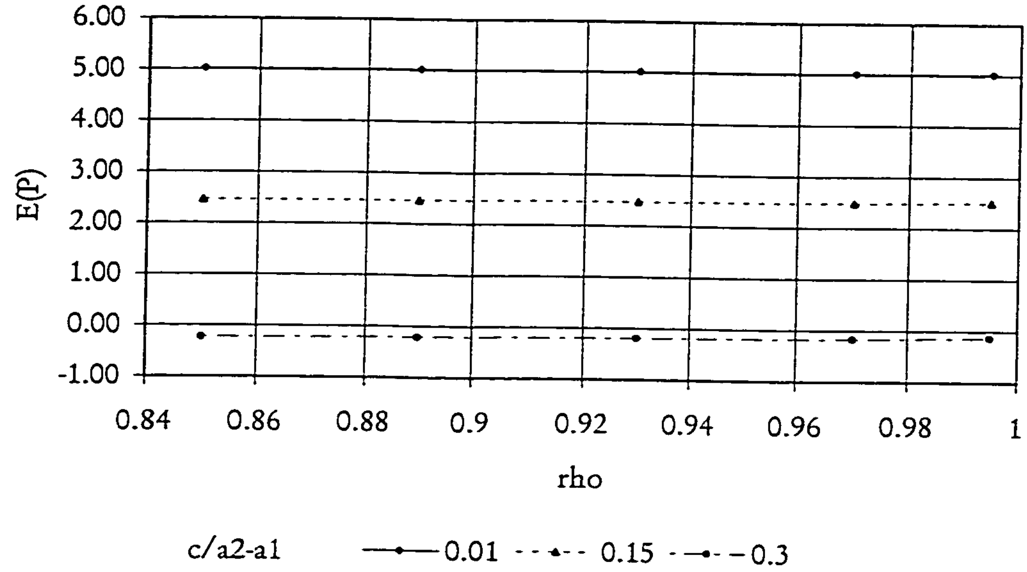


Figure 5-8: E(p) versus measurment error (rho) at a2-r/a1-a2 = 6.5
sigma = 1.75
Special case I

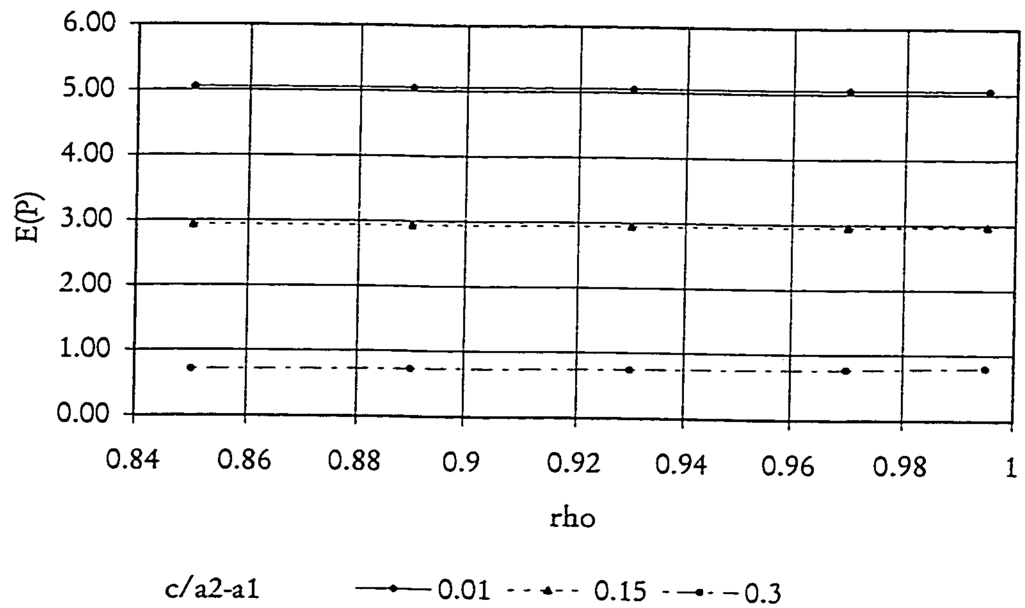


Figure 5-9: E(p) versus measurment error (rho) at a2-r/a1-a2 = 8
sigma = 1.75
Special case I

5.3.2.2 EFFECTS ON OPTIMAL MEAN (SPECIAL CASE I)

In Table 5-2, the optimal means obtained, for the same set of problems are presented. The results show that at a given level of $\frac{c}{a_1 - a_2}, \frac{a_2 - r}{a_1 - a_2}$ & σ_y, the mean tends to decrease. This decrease becomes sharper as $\rho \rightarrow 1$. This shows that, the process mean is forced higher at higher ρ, as the probability of misclassification is higher. These results are shown graphically in figures 5-10 to 5-15. Each figure shows process mean versus measurement error (represented by ρ) containing three plots at three different levels of cost represented by $\frac{c}{a_1 - a_2}$. Process mean decreases sharply with cost as can be seen from the figure 5-10 to 5-15. In each figure the level of $\frac{a_2 - r}{a_1 - a_2}$ and σ_y is fixed. The increase in mean with $\frac{a_2 - r}{a_1 - a_2}$ can be seen from figures 5-10 to 3-12 & figures 5-13 to 5-15 at two different levels of σ_y. By comparing these two sets, it is clear that level of mean increases with the increase in variance of the process.

The results can be summarized as follows:

1. As compared to model 2, the effect of uniformity penalty on the mean is negligible. This is because the target is set at the mean and the products falling in the scrape region is minimal. Therefore, the change in mean will not going effect the penalty due to inconsistency.

TABLE 5-2: Optimal Mean at different parameter settings

(Special case I)

σ_y	$\frac{a_2 - r}{a_1 - a_2}$	ρ	$\frac{c}{a_1 - a_2}$ = 0.01	0.15	0.3
1.25	4	0.85	45.56441	44.10676	43.50375
		0.89	45.50212	44.00292	43.39507
		0.93	45.40617	43.86946	43.26425
		0.97	45.23919	43.67537	43.08347
		0.995	45.00278	43.44076	42.87157
	6	0.85	45.67775	44.23439	43.65026
		0.89	45.60259	44.11931	43.53841
		0.93	45.48969	43.97506	43.40721
		0.97	45.29819	43.77141	43.22906
		0.995	45.03432	43.53165	43.02144
	8	0.85	45.72919	44.29196	43.71650
		0.89	45.64807	44.17212	43.60351
		0.93	45.52745	44.02361	43.47248
		0.97	45.32503	43.81689	43.29597
		0.995	45.04959	43.57659	43.09083
1.75	4	0.85	46.99885	44.89930	44.07290
		0.89	46.91099	44.77147	43.94999
		0.93	46.77640	44.60666	43.79527
		0.97	46.54348	44.36352	43.57146
		0.995	46.21156	44.06400	43.30265
	6	0.85	47.15988	45.09795	44.32662
		0.89	47.05417	44.96368	44.20913
		0.93	46.89763	44.79608	44.06326
		0.97	46.63603	44.55505	43.85192
		0.995	46.27523	44.26116	43.59402
	8	0.85	47.23282	45.18773	44.43953
		0.89	47.11896	45.05108	44.32418
		0.93	46.95262	44.88293	44.18172
		0.97	46.67882	44.64384	43.97552
		0.995	46.30699	44.35362	43.72242

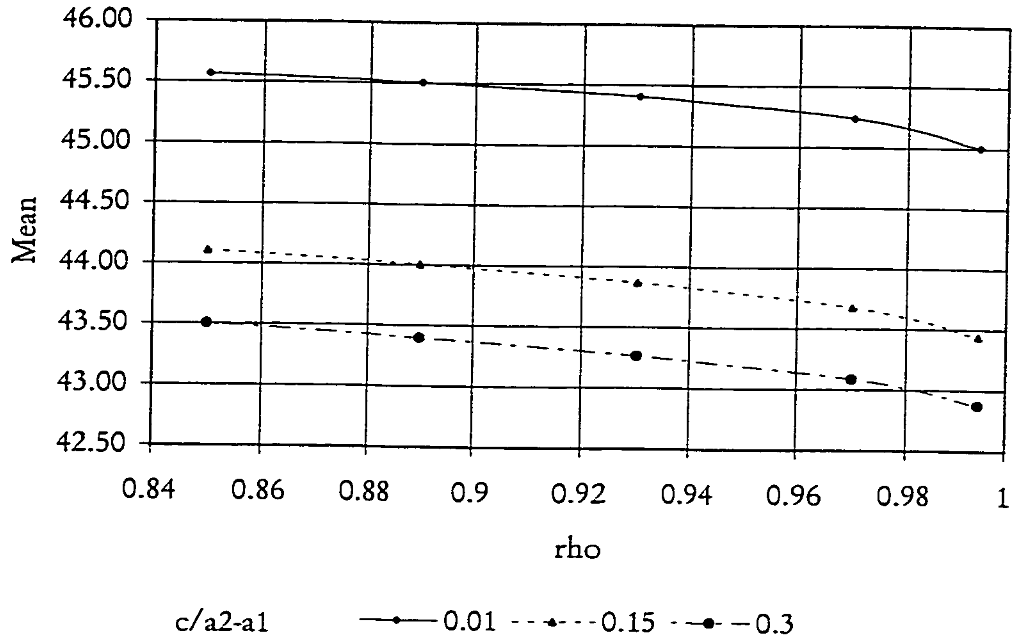


Figure 5-10: Mu versus measurment error (rho) at a2-r/a1-a2 = 4
sigma = 1.25
Special case I

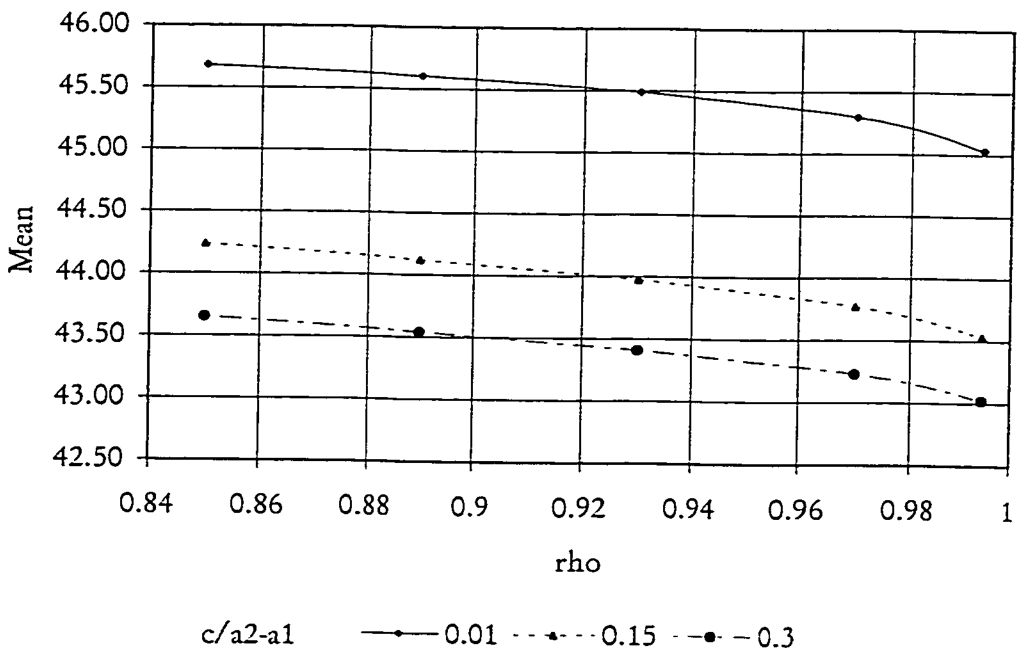


Figure 5-11: Mu versus measurment error (rho) at a2-r/a1-a2 = 6.5
sigma = 1.25
Special case I

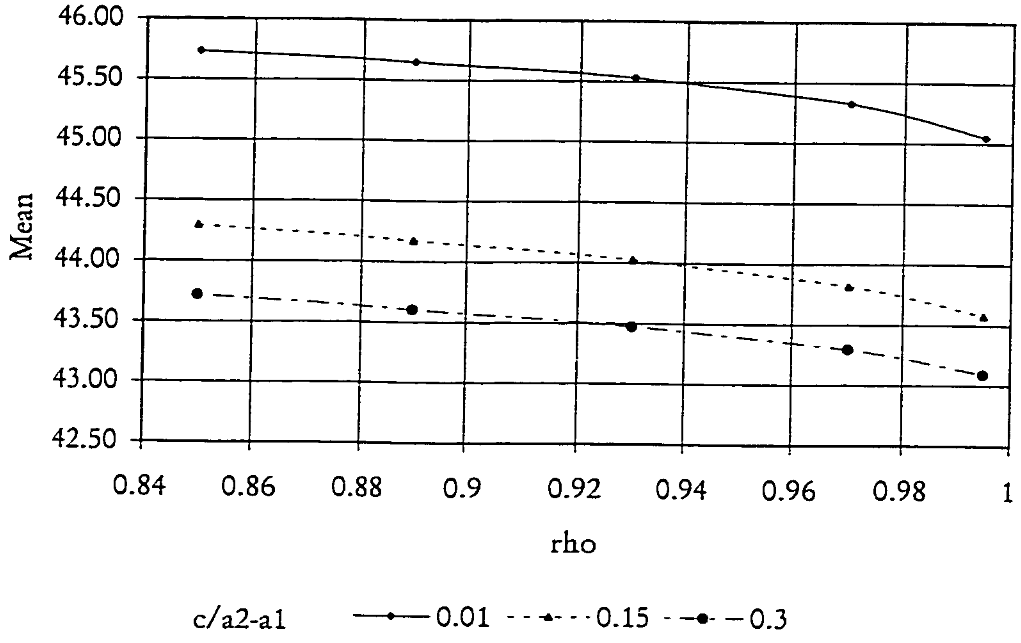


Figure 5-12: Mu versus measurment error (rho) at a2-r/a1-a2 = 8
sigma = 1.25
Special case I

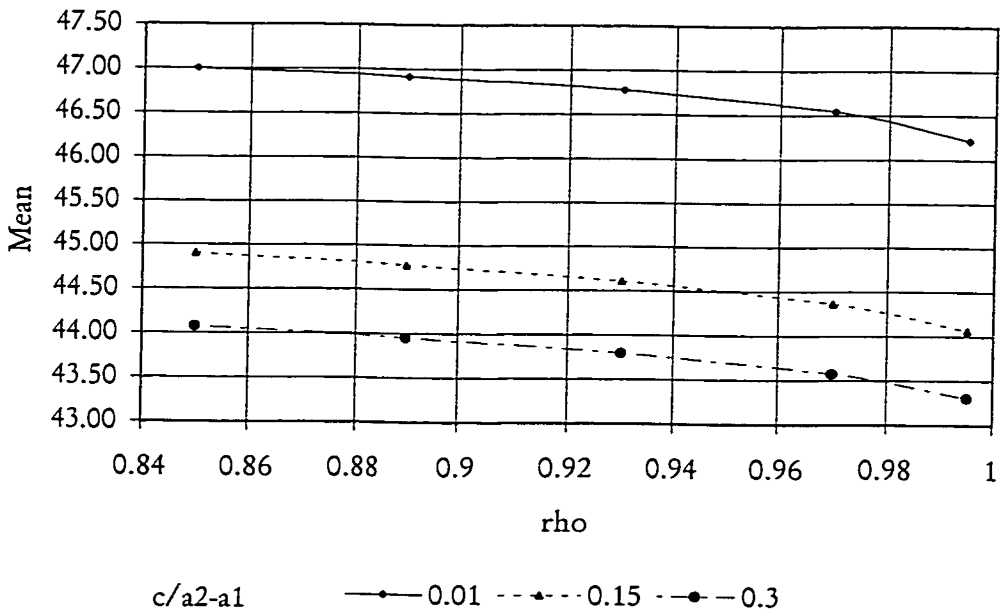


Figure 5-13: Mu versus measurment error (rho) at a2-r/a1-a2 = 4
sigma = 1.75
Special case I

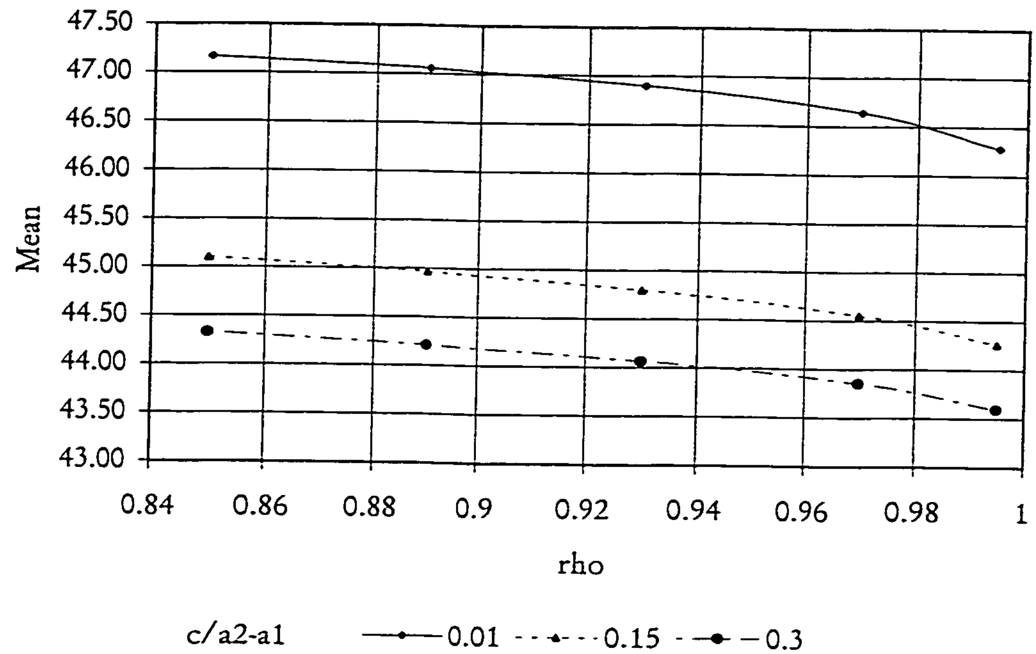


Figure 5-14: Mu versus measurment error (rho) at a2-r/a1-a2 = 6.5
sigma = 1.75
Special case I

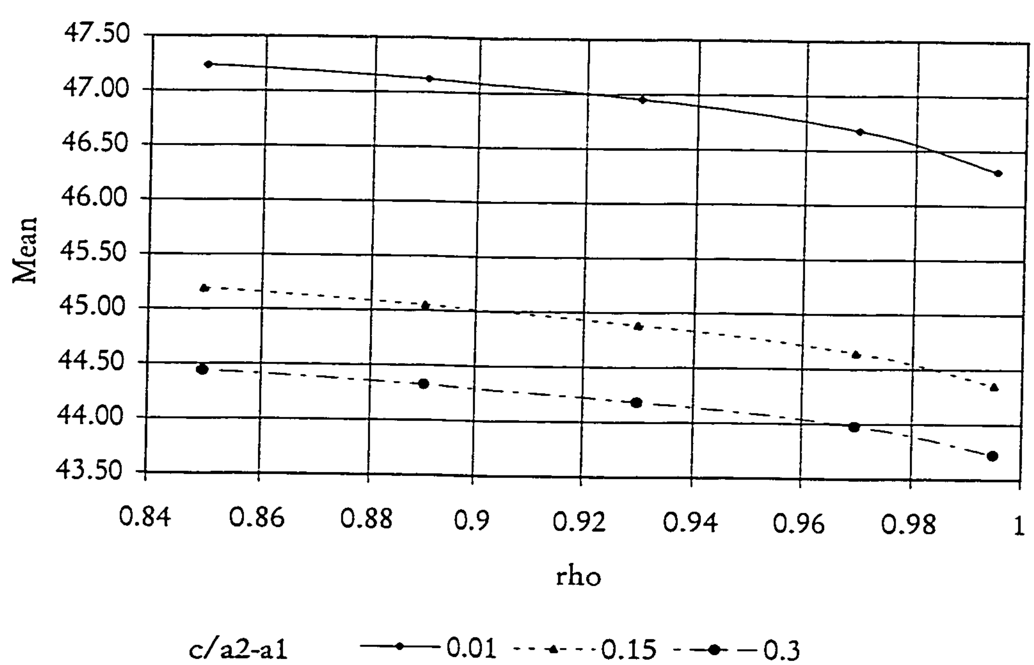


Figure 5-15: Mu versus measurment error (rho) at a2-r/a1-a2 = 8
sigma = 1.75
Special case I

2. The optimal mean decreases sharply with increase in cost. As the term $-c\mu$ in the model shows, the per unit cost is proportional to the product of production cost and the mean of the process. Consequently, the level of mean goes down with the increase in production cost.

3. The optimal mean increase with the increase in $\frac{a_2 - r}{a_1 - a_2}$. This result is due to the reason that as a_2 gets closer to a_1 it will get more profitable (at any fixed cost) and the mean will set higher as a result.

4. The change in optimal mean is less with $\frac{a_2 - r}{a_1 - a_2}$ compared to $\frac{c}{a_1 - a_2}$. This because cost is proportional to process mean while the selling prices are related to the probabilities of the product falling in respective grades. The effect of change in both the parameters will only be closer to each other if the units of the mean are proportional to the probabilities i.e., $\mu > 1$

The optimal mean decrease with the decrease in measurement error. As discussed above, the optimal process mean will be forced higher at higher error, as this will reduce the number of product falling near the specification limit.

5.3.2.3 EFFECTS ON OPTIMAL CUT OFF POINTS (SPECIAL CASE I)

Like in model 2, the effect on cut off points, by varying the parameters, is very important in evaluating the model performance. The location of cut off points determines the effect of measurement error. The value of the cut off point depends mostly upon the loss in profit (grade 1 sold as grade 2) and the penalty imposed if the lower grade classified as higher grade (warranty cost, loss of good will etc.) position of the mean (how close it is to the specification limit, which in turn will determine the probability of misclassification).

The optimal cut off values, are shown in Table 5-3. The penalties are taken at a fixed level and are set at a high value. The results show that both the cut of points are converging to the respective specification limit as $\rho \rightarrow 1$ see figures 5-16 to 5-21. As the mean is set quite higher than the L_2 the number of items misclassifying near L_2 will be significantly less. On the other hand, the chances of items misclassified around L_1 are substantially higher as compared to L_2 so the main emphasis is kept on w_1.

The figure shows that the cut off point w_1, initially increase then decrease, especially at lower costs, with the decrease in measurement error. This initial rise is because the optimal mean with lowering error gets closer to the specification limits (as shown in figures 5-16 to 5-21) and showing an increase in probability of misclassification. Consequently, with high penalties, the cut off point w_1 tend to increase initially, but as $\rho \rightarrow 1$ the error reduces as a result, the w_1 starts getting closer to the specification limit L_1 again. Another noticeable thing is that the w_1 starts initially lower than L_1 then crosses it over and finally starts bending again towards the specification limit L_1. The point, during this rise and fall, at which the value of w_1 becomes equal to the L_1 shows that, at that level of error the model manages to find a location where

TABLE 5-3: Optimal cut off points at different parameter settings

(Special case I)

			$\frac{c}{a_1 - a_2}$ = 0.01		0.15		0.3	
σ_y	$\frac{a_2 - r}{a_1 - a_2}$	ρ	w_1	w_2	w_1	w_2	w_1	w_2
1.25	4	0.85	41.0086	37.9084	41.5685	38.4684	41.8002	38.7031
		0.89	41.2948	38.5924	41.6883	38.9855	41.8476	39.1461
		0.93	41.5140	39.1822	41.7540	39.4223	41.8486	39.5170
		0.97	41.6446	39.6871	41.7428	39.7854	41.7800	39.8225
		0.995	41.6129	39.9558	41.6286	39.9717	41.6343	39.9774
	6	0.85	41.1753	37.7744	41.7297	38.3279	41.9539	38.5548
		0.89	41.4341	38.4951	41.8234	38.8827	41.9757	39.0362
		0.93	41.6232	39.1158	41.8598	39.3523	41.9485	39.4413
		0.97	41.7152	39.6510	41.8111	39.7465	41.8452	39.7810
		0.995	41.6416	39.9427	41.6567	39.9581	41.6619	39.9632
	8	0.85	41.2479	37.7231	41.8000	38.2738	42.0208	38.4974
		0.89	41.4950	38.4589	41.8824	38.8436	42.0315	38.9936
		0.93	41.6710	39.0913	41.9059	39.3258	41.9921	39.4124
		0.97	41.7462	39.6377	41.8409	39.7319	41.8737	39.7654
		0.995	41.6542	39.9380	41.6690	39.9532	41.6739	39.9581
1.75	4	0.85	40.8721	37.3770	41.6786	38.1822	41.9960	38.4997
		0.89	41.2583	38.2370	41.8198	38.7980	42.0354	39.0137
		0.93	41.5489	38.9788	41.8878	39.3175	42.0145	39.4441
		0.97	41.7145	39.6117	41.8514	39.7486	41.9012	39.7983
		0.995	41.6600	39.9462	41.6816	39.9679	41.6893	39.9756
	6	0.85	41.1094	37.1884	41.9013	37.9777	42.1975	38.2736
		0.89	41.4545	38.0980	42.0032	38.6463	42.2011	38.8437
		0.93	41.7014	38.8853	42.0296	39.2129	42.1441	39.3280
		0.97	41.8128	39.5605	41.9434	39.6912	41.9876	39.7354
		0.995	41.6999	39.9277	41.7202	39.9482	41.7270	39.9549
	8	0.85	41.2122	37.1164	41.9976	37.8982	42.2850	38.1849
		0.89	41.5400	38.0455	42.0828	38.5877	42.2737	38.7789
		0.93	41.7682	38.8505	42.0914	39.1729	42.2010	39.2833
		0.97	41.8558	39.5419	41.9836	39.6697	42.0256	39.7117
		0.995	41.7175	39.9211	41.7372	39.9411	41.7435	39.9474

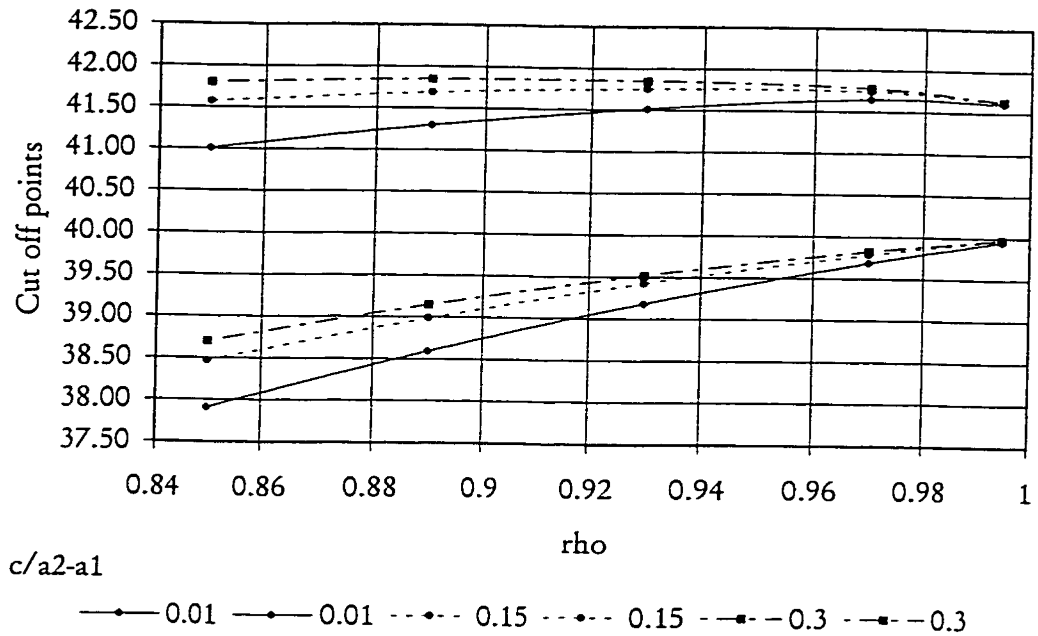


Figure 5-16: w1, w2 versus measurment error (rho) at
a2-r/a1-a2 = 4, sigma = 1.25
Special case I

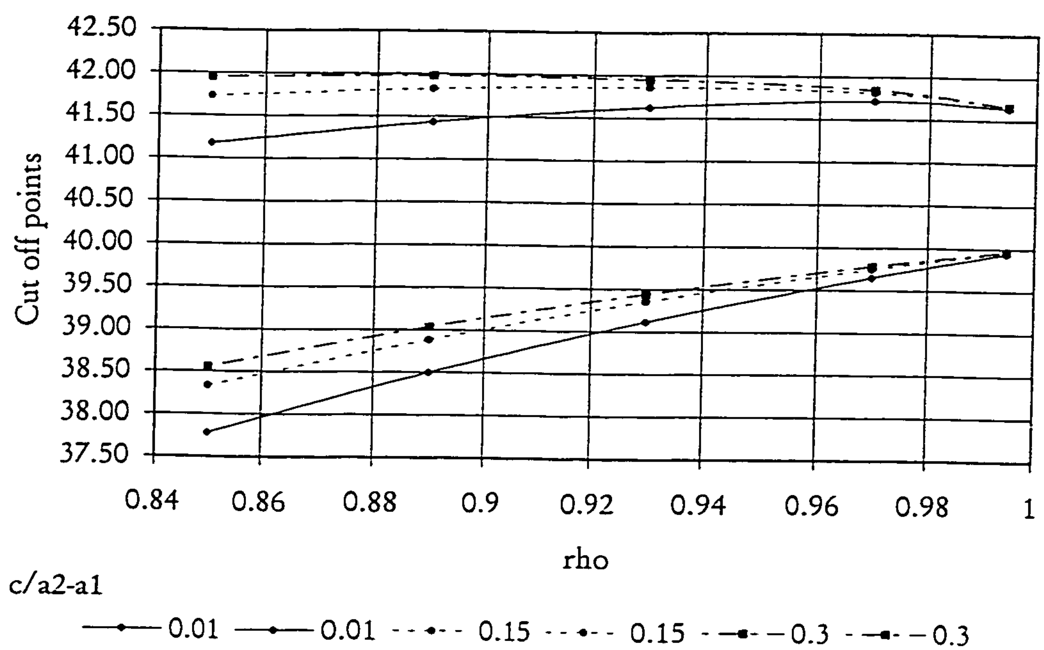


Figure 5-17: w1, w2 versus measurment error (rho) at
a2-r/a1-a2 = 6.5, sigma = 1.25
Special case I

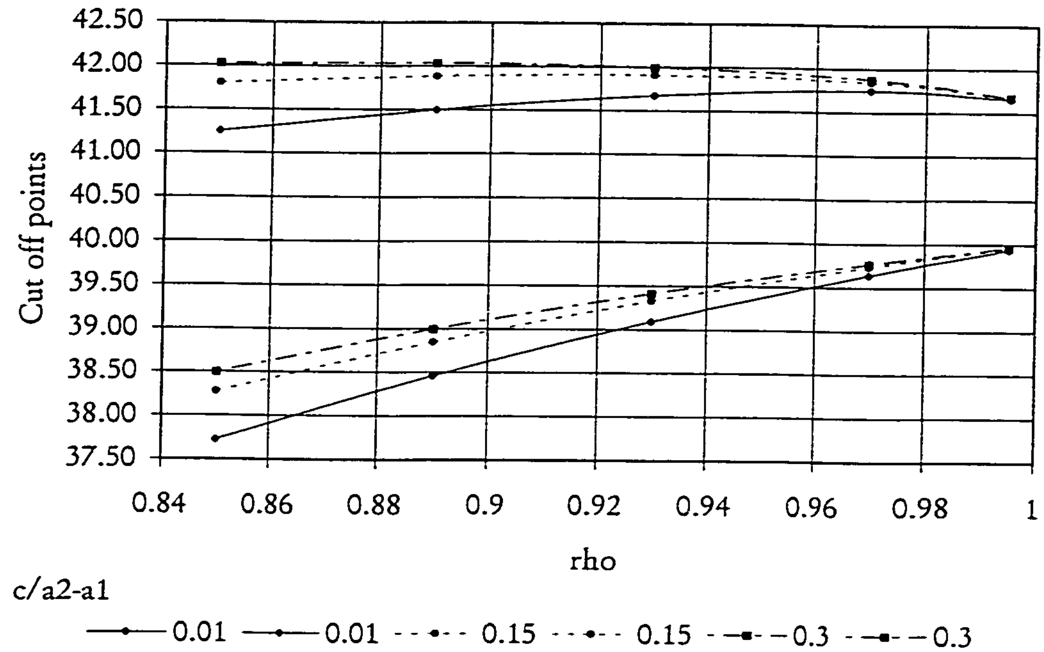


Figure 5-18: w1, w2 versus measurment error (rho) at
a2-r/a1-a2 = 8, sigma = 1.25
Special case I

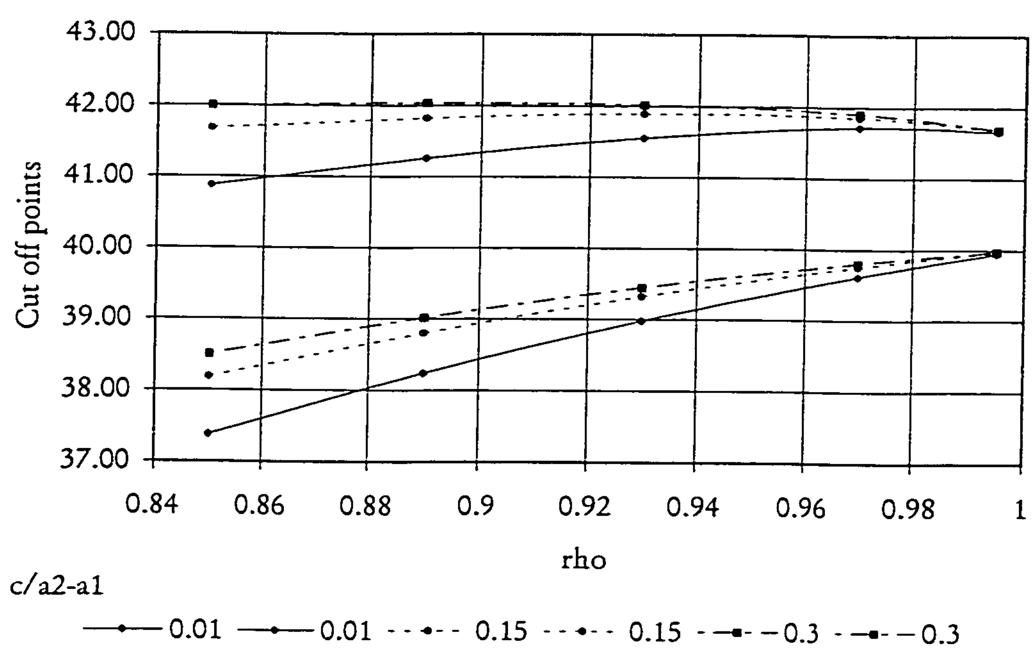


Figure 5-19: w1, w2 versus measurment error (rho) at
a2-r/a1-a2 = 4, sigma = 1.75
Special case I

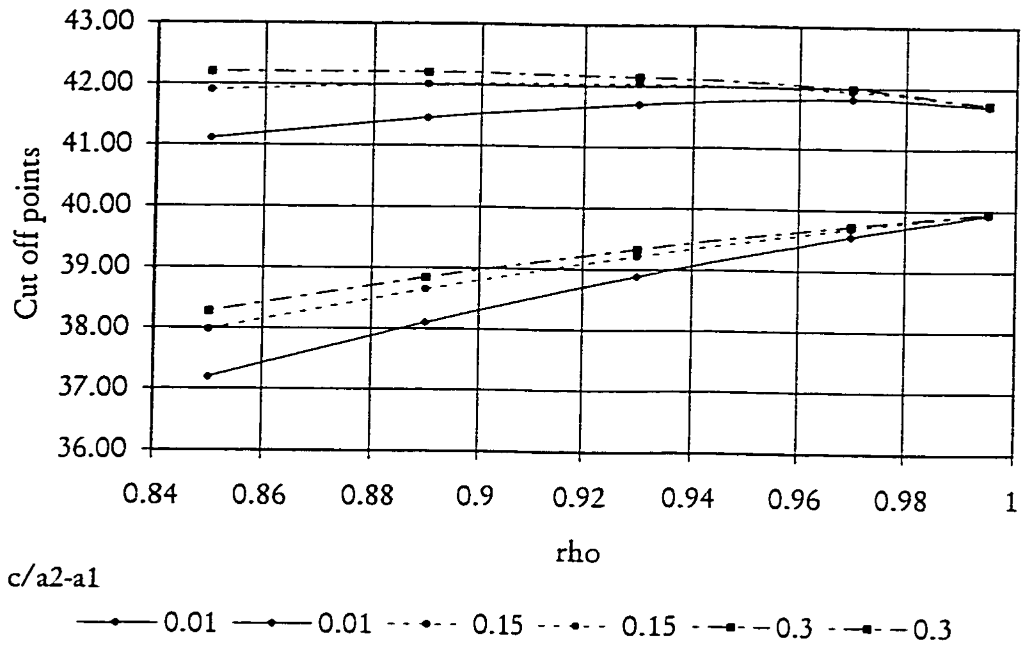


Figure 5-20: w1, w2 versus measurment error (rho) at
a2-r/a1-a2 = 6.5, sigma = 1.75
Special case I

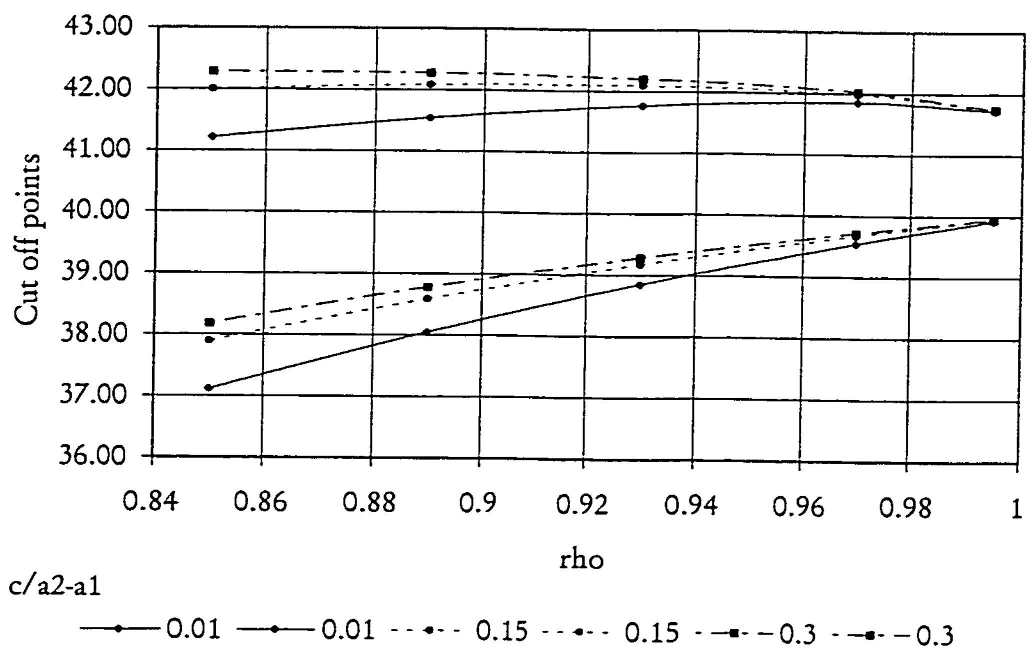


Figure 5-21: w1, w2 versus measurment error (rho) at
a2-r/a1-a2 = 8, sigma = 1.75
Special case I

the expected loss of the misclassification on either side becomes the same.

The above discussion can be summarized as follows:

1. As compared to model 2, the cut off points are not affected by non-uniformity penalty. This is intuitive as the cut off points are dependant on the number of items misclassifying which is determined by the location of the process mean. Since the process mean is not much affected the cut off points are also similar to as in model 2.
2. Both the cut of points are converging to the respective specification limit as $\rho \to 1$.
3. The changes in cut off points with measurement error are higher at lower cost because of rapid changes in mean at lower costs (as shown in figures 5-16 to 5-21).
4. The variation in w_2 is higher than w_1. This is because the mean is far from w_2 and the effect of changing, w_2 is less as compared to w_1.
5. At higher level of costs, the results tend to remain conservative, (both cut off point tend to remain higher) i.e., it is better to lose profit than paying the penalty.

5.3.2.5 EFFECTS ON EXPECTED PROFIT (SPECIAL CASE II)

In Table 5-4, the expected profit at different parameter levels is shown. The results show that at a given level of $\frac{c}{a_1 - a_2}, \frac{a_2 - r}{a_1 - a_2}$ & σ_y while k is taken at 0.05. As compared special case I the effect of non-uniformity on the expected profit is higher even at very low value of k = 0.05. This response was expected as target value, unlike special case I, is fixed and is not related to the mean. Therefore, as much as the optimal process differs form the target the effect will be more. The results can be shows graphically in figures 5-22 to 5-27.

Each figure shows per unit expected profit versus measurement error (represented by ρ) containing three plots at three different levels of cost represented by $\frac{c}{a_1 - a_2}$. Per unit, expected profit decreases sharply with cost as can be seen from the figures 5-22 to 5-27. In each figure the level of $\frac{a_2 - r}{a_1 - a_2}$ and σ_y is fixed. The variation in per unit expected profit with $\frac{a_2 - r}{a_1 - a_2}$ at given level of σ_y (two level), as can be seen by comparing figures 5-22 to 5-24 and figures 5-25 to 5-27, is less as compared to $\frac{c}{a_1 - a_2}$.

The results can be summarized as follows:

1. The effect of non-uniformity penalty on the expected mean is significant as compared to special case I. This is due to the fact that, the target value is now fixed at L_1 and is not taken at mean. In turn as the mean goes off this target value the effect of non-uniformity penalty will become pronounce.
2. The model performed very well in nullifying the effect of error in measurement (at any given level of $\frac{c}{a_1 - a_2}, \frac{a_2 - r}{a_1 - a_2}$ & σ_y). As the level of profit is almost maintained at different levels of measurement error.

TABLE 5-4: Expected profit at different parameter settings

(Special case II)

σ_y	$\frac{a_2 - r}{a_1 - a_2}$	rho	$\frac{c}{a_1 - a_2}$ 0.01	0.15	0.3
1.25	4	0.85	4.68683	1.05162	-2.81766
		0.89	4.70568	1.07617	-2.78739
		0.93	4.72901	1.10641	-2.75007
		0.97	4.76160	1.14873	-2.69756
		0.995	4.79903	1.19781	-2.63609
	6	0.85	4.80841	2.39232	-0.18386
		0.89	4.82681	2.41374	-0.15959
		0.93	4.84890	2.43961	-0.12998
		0.97	4.87898	2.47521	-0.08869
		0.995	4.91287	2.51595	-0.04070
	8	0.85	4.85529	2.86477	0.74074
		0.89	4.87281	2.88442	0.76236
		0.93	4.89373	2.90808	0.78875
		0.97	4.92206	2.94057	0.82555
		0.995	4.95392	2.97772	0.86830
1.75	4	0.85	4.42920	0.76626	-3.12884
		0.89	4.45567	0.79861	-3.09023
		0.93	4.48900	0.83951	-3.04113
		0.97	4.53648	0.89816	-2.97027
		0.995	4.59207	0.96743	-2.88600
	6	0.85	4.56466	2.12839	-0.46842
		0.89	4.58860	2.15538	-0.43823
		0.93	4.61840	2.18931	-0.39987
		0.97	4.66058	2.23787	-0.34439
		0.995	4.70993	2.29531	-0.27814
	8	0.85	4.61614	2.60812	0.46588
		0.89	4.63841	2.63261	0.49271
		0.93	4.66619	2.66348	0.52691
		0.97	4.70564	2.70783	0.57657
		0.995	4.75199	2.76050	0.63611

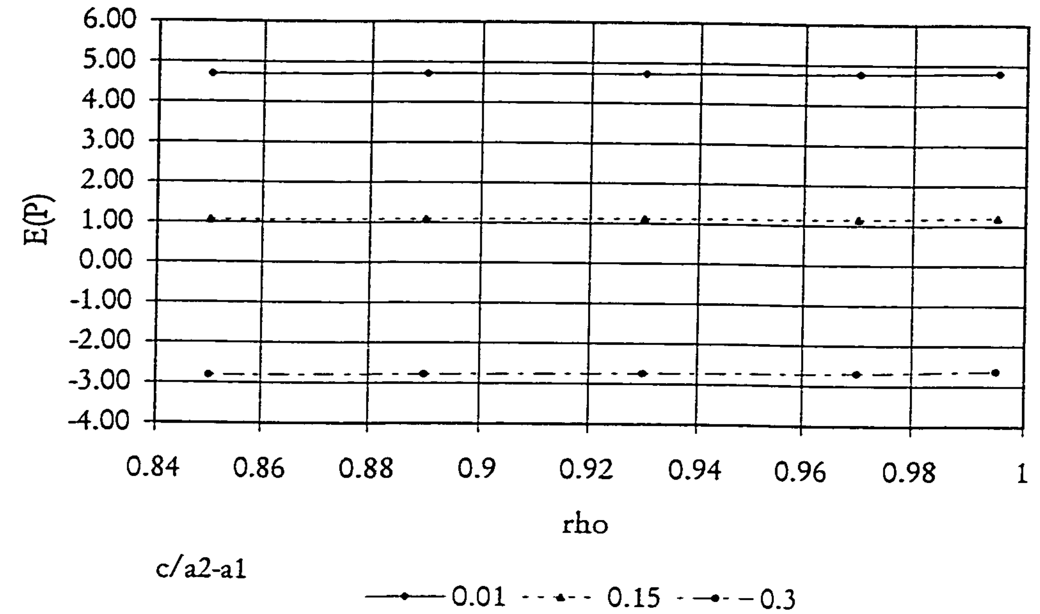


Figure 5-22: E(p) versus measurment error (rho) at a2-r/a1-a2 = 4
sigma = 1.25
Special case II

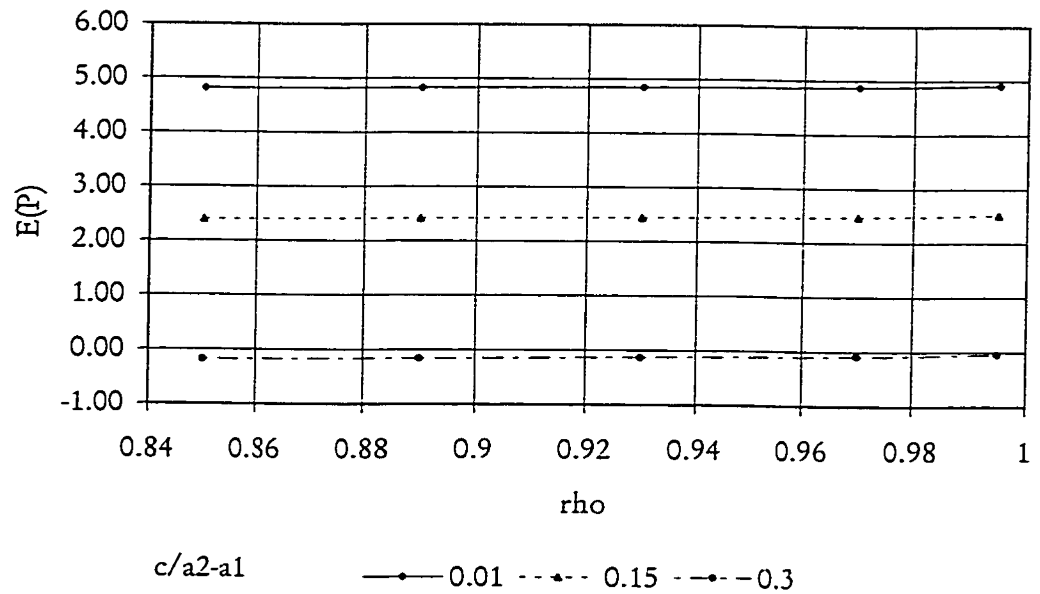


Figure 5-23: E(p) versus measurment error (rho) at
a2-r/a1-a2 = 6.5, sigma = 1.25
Special case II

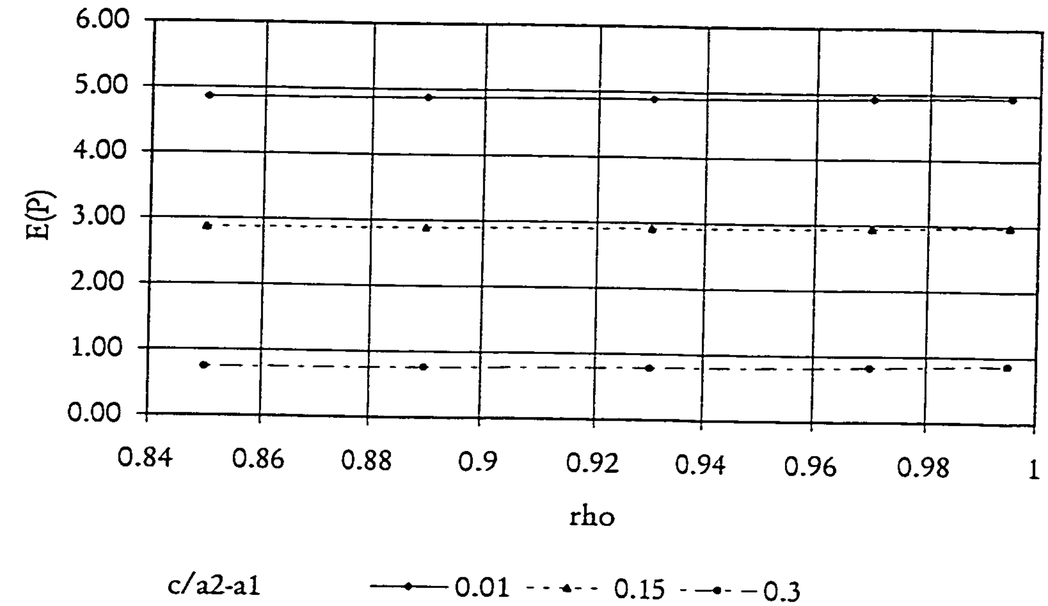


Figure 5-24: E(p) versus measurment error (rho) at a2-r/a1-a2 = 8
sigma = 1.25
Special case II

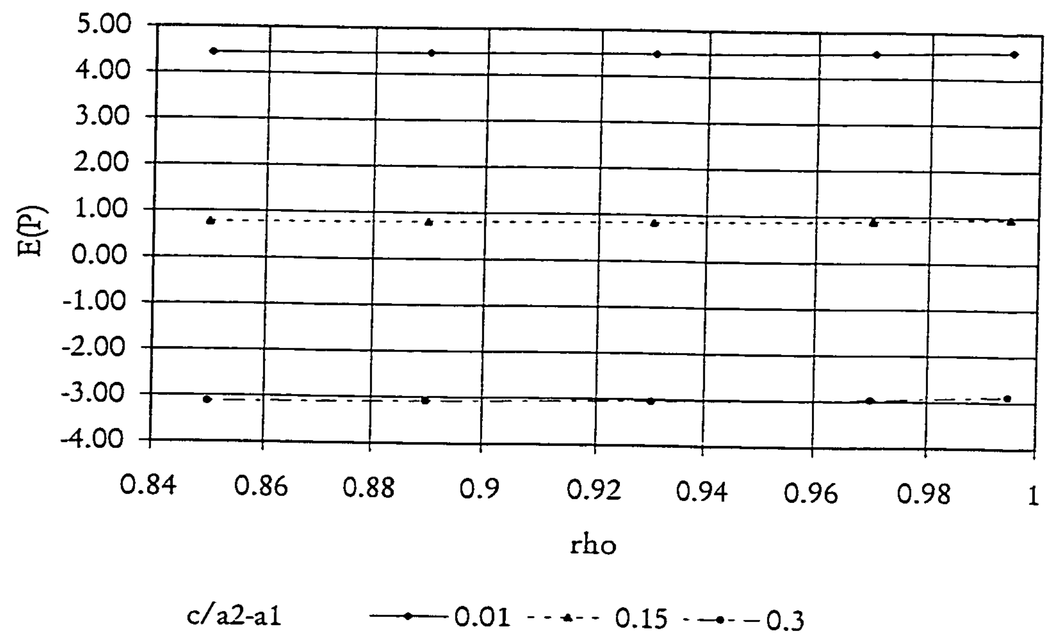


Figure 5-25 E(p) versus measurment error (rho) at a2-r/a1-a2 = 4
sigma = 1.75
Special case II

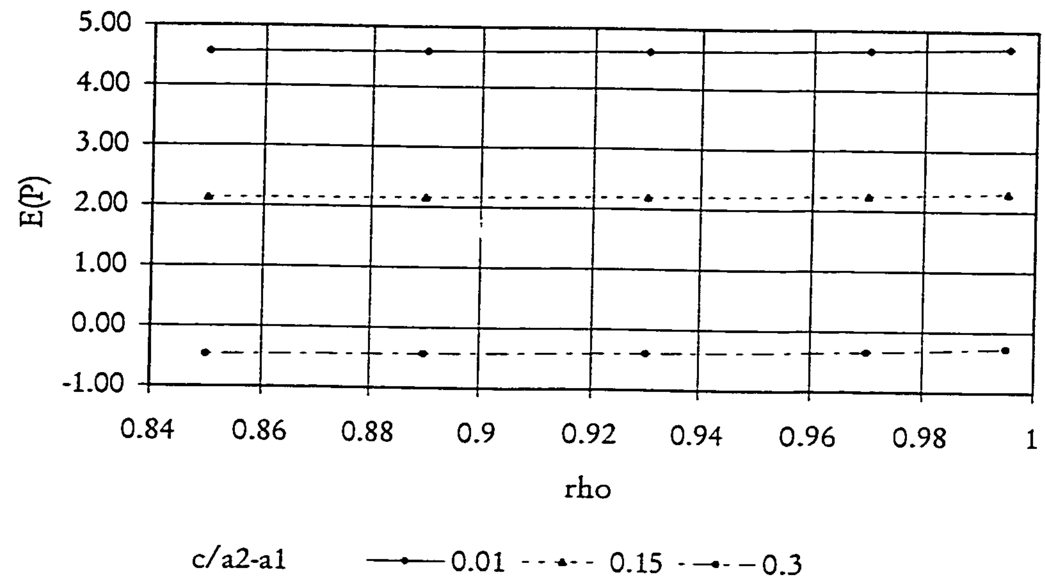


Figure 5-26: E(p) versus measurment error (rho) at a2-r/a1-a2 = 6.5
sigma = 1.75
Special case II

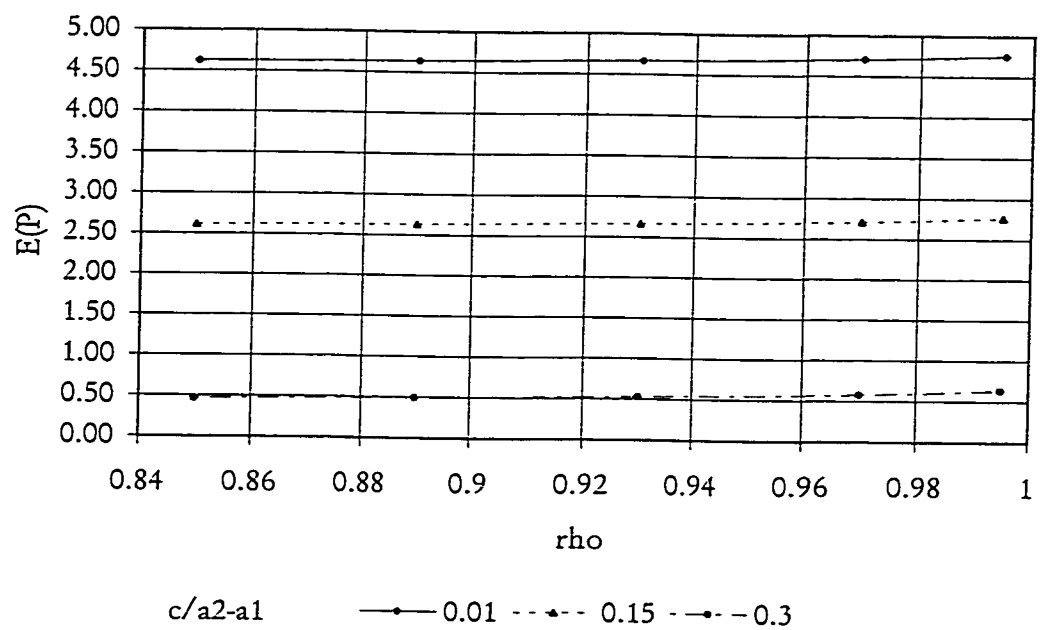


Figure 5-27: E(p) versus measurment error (rho) at a2-r/a1-a2 = 8
sigma = 1.75
Special case II

3. $\frac{c}{a_1 - a_2}$, significantly reduces per unit expected profit as expected (cost of production is proportional to the level of the process mean).

4. The effect of increase in $\frac{a_2 - r}{a_1 - a_1}$ is less on per expected profit, as compared to $\frac{c}{a_1 - a_2}$. This is because the cost is directly related to mean while the selling prices are proportional to the probability of the product falling in the respective grade.

5.3.2.6 EFFECTS ON OPTIMAL MEAN (SPECIAL CASE II)

Table 5-5, the optimal means obtained, for the same set of problems are presented. The results show that, as compared to special case I the mean is significantly reduced, the reason behind this phenomenon is that the target is set at L_1 and the mean, in order to reduce the non-uniformity penalty, is forced to pull closer to L_1. The mean also show a decrease with error, this decrease becomes sharper as $\rho \rightarrow 1$. This shows that, the process mean is forced higher at higher ρ, as the probability of misclassification is higher. These results are shown graphically in figures 5-28 to 5-33. Each figure shows process mean versus measurement error (represented by ρ) containing three plots at three different levels of cost represented by $\frac{c}{a_1 - a_2}$. The general behavior of the change in mean is similar to as special case I.

The results can be summarized as follows:

TABLE 5-5: Optimal Mean at different parameter settings

(Special case II)

σ_y	$\frac{a_2 - r}{a_1 - a_2}$	rho	$\frac{c}{a_1 - a_2}$ 0.01	0.15	0.3
1.25	4	0.85	43.41988	43.13592	42.85060
		0.89	43.35136	43.06936	42.79067
		0.93	43.26844	42.98837	42.71492
		0.97	43.15255	42.87333	42.60399
		0.995	43.01489	42.73467	42.46767
	6	0.85	43.25022	43.04074	42.83419
		0.89	43.19347	42.98992	42.79033
		0.93	43.12376	42.92512	42.73071
		0.97	43.02365	42.82830	42.63781
		0.995	42.90115	42.70657	42.51804
	8	0.85	43.17386	42.99736	42.82396
		0.89	43.12524	42.95452	42.78693
		0.93	43.06355	42.89745	42.73407
		0.97	42.97209	42.80903	42.64879
		0.995	42.85738	42.69507	42.53623
1.75	4	0.85	43.76949	43.44611	43.11309
		0.89	43.69954	43.37624	43.04403
		0.93	43.61016	43.28549	42.95270
		0.97	43.47905	43.15095	42.81599
		0.995	43.31877	42.98587	42.64824
	6	0.85	43.61695	43.39458	43.16742
		0.89	43.56178	43.34070	43.11458
		0.93	43.48789	43.26693	43.04073
		0.97	43.37493	43.15235	42.92476
		0.995	43.23204	43.00655	42.77678
	8	0.85	43.55616	43.37233	43.18421
		0.89	43.50763	43.32475	43.13725
		0.93	43.44062	43.25764	43.06984
		0.97	43.33546	43.15108	42.96186
		0.995	43.20012	43.01321	42.82186

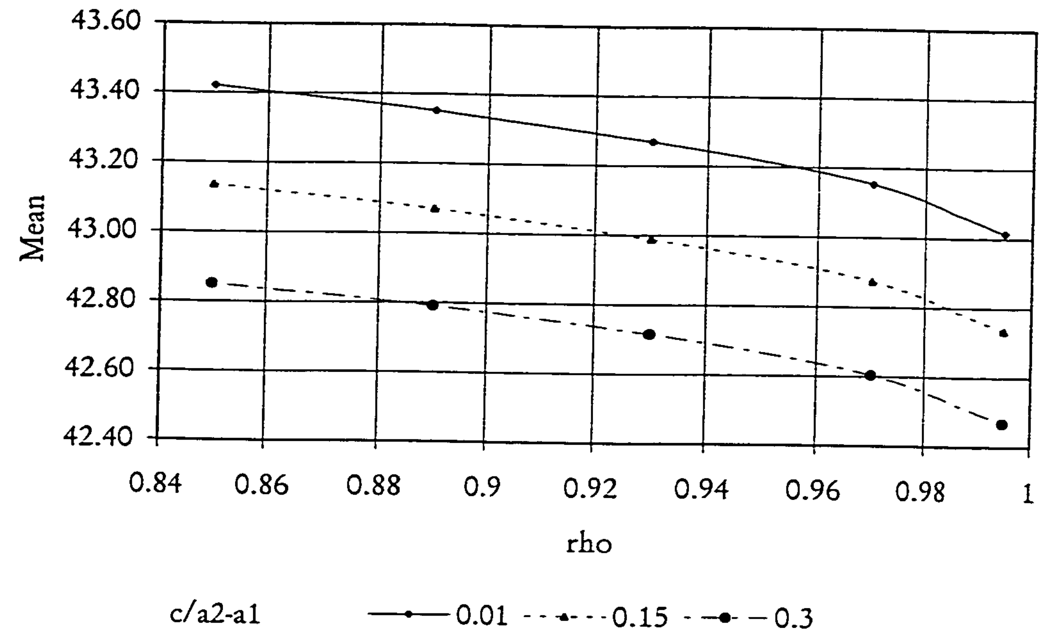


Figure 5-28: Mu versus measurment error (rho) at a2-r/a1-a2 = 4
sigma = 1.25
Special case II

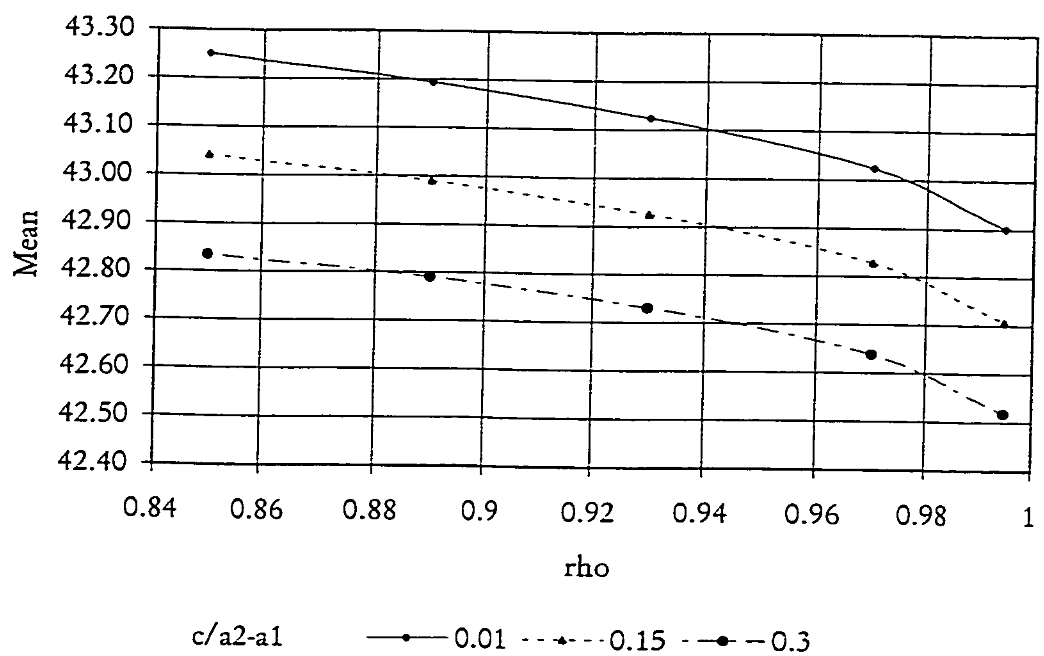
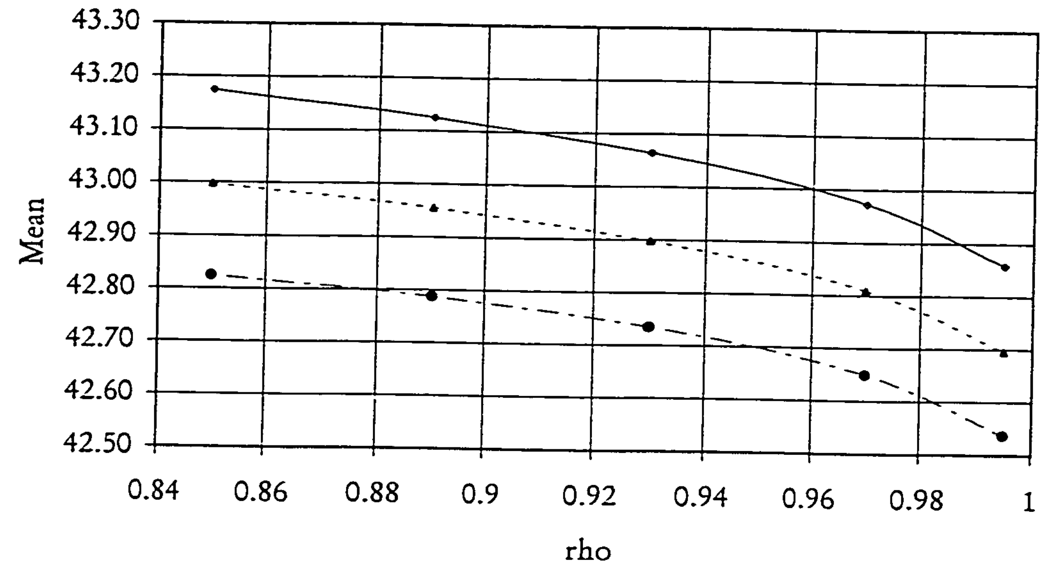
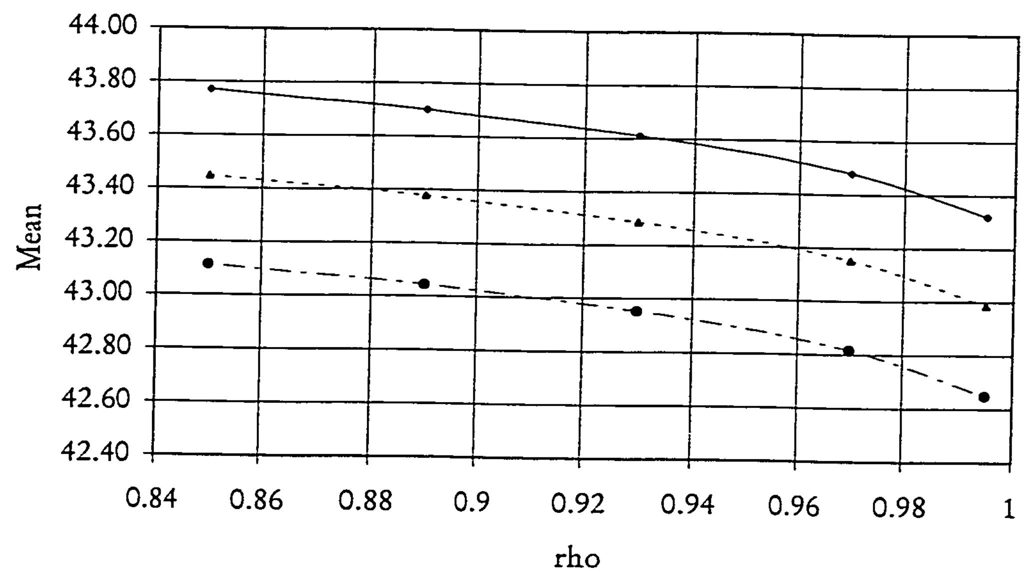


Figure 5-29: Mu versus measurment error (rho) at a2-r/a1-a2 = 6.5
sigma = 1.25
Special case II



c/a2-a1 —◆—0.01 - - ▲- - -0.15 - —●- —0.3

Figure 5-30: Mu versus measurment error (rho) at a2-r/a1-a2 = 8
sigma = 1.25
Special case II



c/a2-a1 —◆—0.01 - - ▲- - -0.15 - —●- —0.3

Figure 5-31: Mu versus measurment error (rho) at a2-r/a1-a2 = 4
sigma = 1.75
Special case II

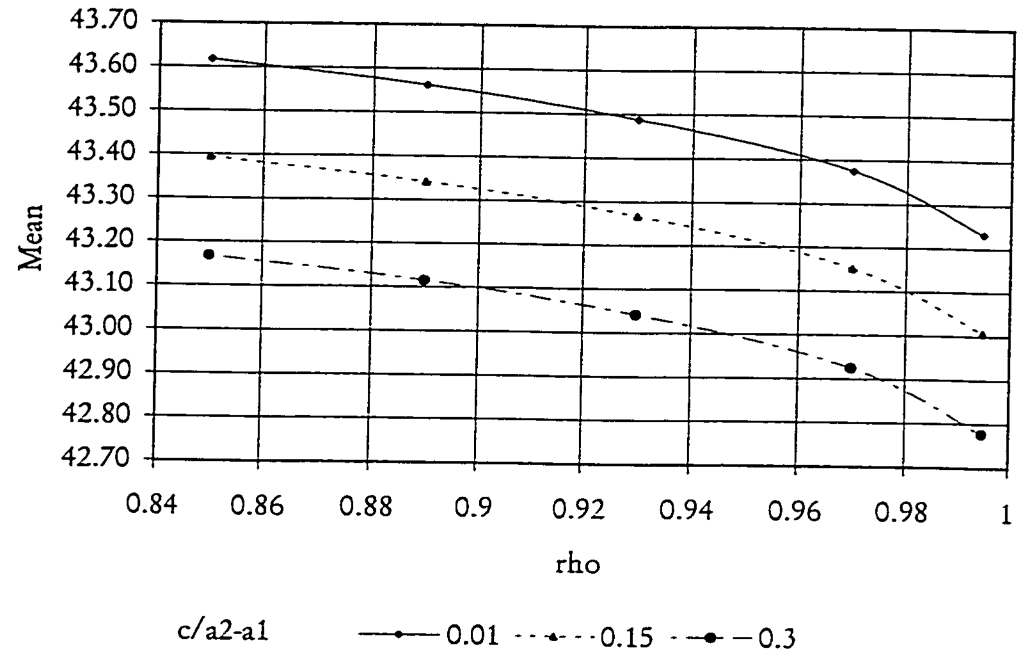


Figure 5-32: Mu versus measurment error (rho) at a2-r/a1-a2 = 6.5
sigma = 1.75
Special case II

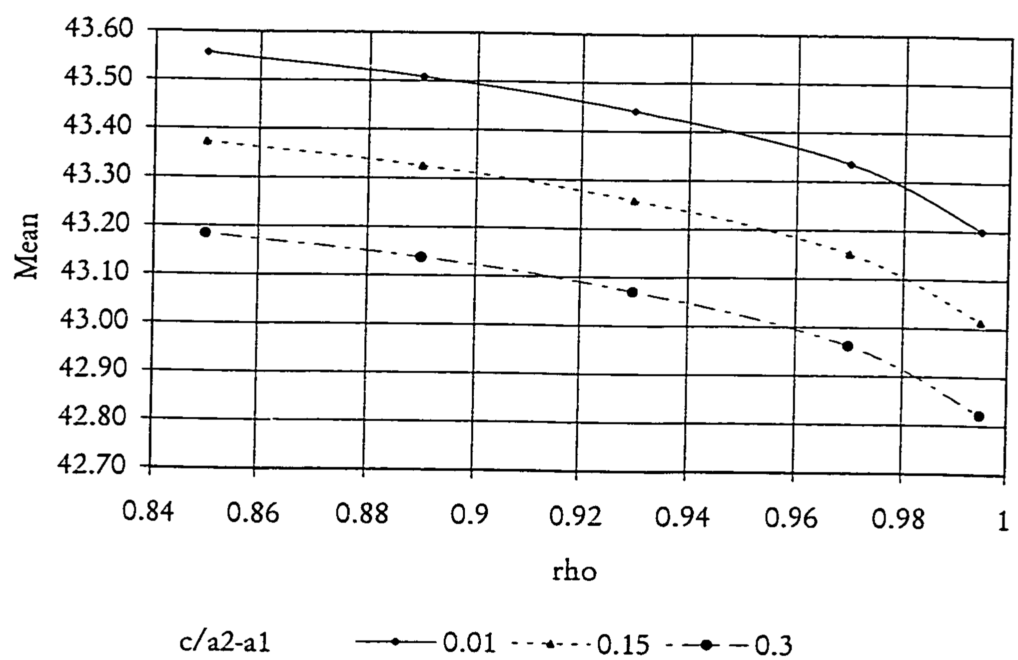


Figure 5-33: Mu versus measurment error (rho) at a2-r/a1-a2 = 8
sigma = 1.75
Special case II

1. As compared to model 2 and special case I, the effect of uniformity penalty on the mean is significant. As discussed earlier this is because the target is now at the mean and therefore it is forced to get closer to the target.
2. The optimal mean decreases sharply with increase in cost. As the term -cμ in the model shows, the per unit cost is proportional to the product of production cost and the mean of the process. Consequently, the level of mean goes down with the increase in production cost.
3. The optimal mean increase with the increase in $\frac{a_2 - r}{a_1 - a_1}$. This result is due to the reason that as a_2 gets closer to a_1 it will get more profitable (at any fixed cost) and the mean will set higher as a result.
4. The change in optimal mean is less with $\frac{a_2 - r}{a_1 - a_1}$ compared to $\frac{c}{a_1 - a_2}$. This because cost is proportional to process mean while the selling prices are related to the probabilities of the product falling in respective grades. The effect of change in both the parameters will only be closer to each other if the units of the mean are proportional to the probabilities i.e., $\mu > 1$
5. The optimal mean decrease with the decrease in measurement error. As discussed above, the optimal process mean will be forced higher at higher error, as this will reduce the number of product falling near the specification limit.

5.3.2.7 EFFECTS ON OPTIMAL CUT OFF POINTS (SPECIAL CASE II)

The role of cut off points, is very important in evaluating the model performance as discussed in chapter 2 and special case I. The location of cut off points determines the effect of measurement error.

The optimal cut off values, are shown in Table 5-6. The penalties are taken at a fixed level and are set at a high value. The results show that both the cut of points are converging to the respective specification limit as $\rho \to 1$ see figures 5-34 to 5-39. As the mean is set quite higher than the L_2 the number of items misclassifying near L_2 will be significantly less. On the other hand, the chances of items misclassified around L_1 are substantially higher as compared to L_2 so the main emphasis is kept on w_1.

Unlike model 2 and special case 1, the cut off point w_1, is always kept higher than L_1. This is due the fact that as the optimal mean is forced closer to the specification limit L_1, therefore in order to reduce to effect of error the cut off point w_1 is always set higher than L_1. w_2, on the other hand, is not much affected because the mean is set away from it.

The above discussion can be summarized as follows:

1. As compared to model 2 and special case I, the cut off point w_1 is always kept higher than L_1 because the mean is set close to L_1 and the probability of misclassification is higher.
2. Both the cut of points are converging to the respective specification limit as $\rho \to 1$.

TABLE 5-6: Optimal cut off points at different parameter settings

(Special case II)

			$\frac{c}{a_1 - a_2}$ = 0.01		0.15		0.3	
σ_y	$\frac{a_2 - r}{a_1 - a_2}$	ρ	w_1	w_2	w_1	w_2	w_1	w_2
1.25	4	0.85	41.8324	38.7343	41.9408	38.8456	42.0511	38.9508
		0.89	41.8594	39.1587	41.9328	39.2321	42.0065	39.3036
		0.93	41.8479	39.5162	41.8915	39.5601	41.9344	39.6026
		0.97	41.7757	39.8182	41.7932	39.8357	41.8101	39.8526
		0.995	41.6329	39.9759	41.6357	39.9788	41.6384	39.9815
	6	0.85	42.1073	38.7098	42.1873	38.7921	42.2675	38.8657
		0.89	42.0660	39.1270	42.1193	39.1810	42.1722	39.2315
		0.93	41.9926	39.4856	42.0236	39.5167	42.0542	39.5468
		0.97	41.8581	39.7939	41.8704	39.8062	41.8823	39.8181
		0.995	41.6631	39.9644	41.6650	39.9664	41.6669	39.9683
	8	0.85	42.2288	38.7072	42.2964	38.7765	42.3639	38.8376
		0.89	42.1568	39.1194	42.2014	39.1648	42.2460	39.2071
		0.93	42.0558	39.4763	42.0817	39.5023	42.1074	39.5276
		0.97	41.8941	39.7864	41.9042	39.7960	41.9143	39.8061
		0.995	41.6763	39.9605	41.6779	39.9621	41.6795	39.9637
1.75	4	0.85	42.1125	38.6162	42.2369	38.7413	42.3644	38.8690
		0.89	42.1011	39.0794	42.1861	39.1651	42.2731	39.2517
		0.93	42.0434	39.4731	42.0942	39.5244	42.1462	39.5761
		0.97	41.9069	39.8037	41.9276	39.8246	41.9486	39.8458
		0.995	41.6891	39.9754	41.6925	39.9788	41.6959	39.9823
	6	0.85	42.4699	38.5455	42.5553	38.6308	42.6426	38.7187
		0.89	42.3709	39.0132	42.4288	39.0712	42.4883	39.1312
		0.93	42.2337	39.4163	42.2686	39.4524	42.3039	39.4877
		0.97	42.0175	39.7653	42.0315	39.7793	42.0458	39.7936
		0.995	41.7306	39.9586	41.7329	39.9609	41.7352	39.9632
	8	0.85	42.6238	38.5220	42.6954	38.5949	42.7675	38.6673
		0.89	42.4873	38.9905	42.5361	39.0411	42.5853	39.0904
		0.93	42.3169	39.3990	42.3454	39.4276	42.3747	39.4570
		0.97	42.0658	39.7519	42.0774	39.7635	42.0892	39.7754
		0.995	41.7488	39.9527	41.7507	39.9546	41.7526	39.9565

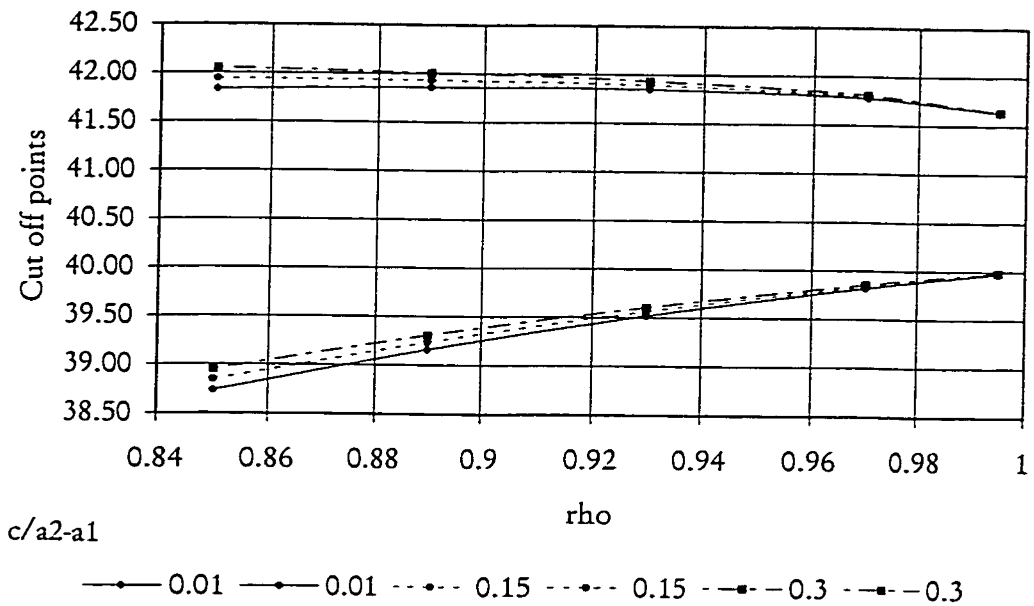


Figure 5-34: w1, w2 versus measurment error (rho) at
a2-r/a1-a2 = 4, sigma = 1.25
Special case II

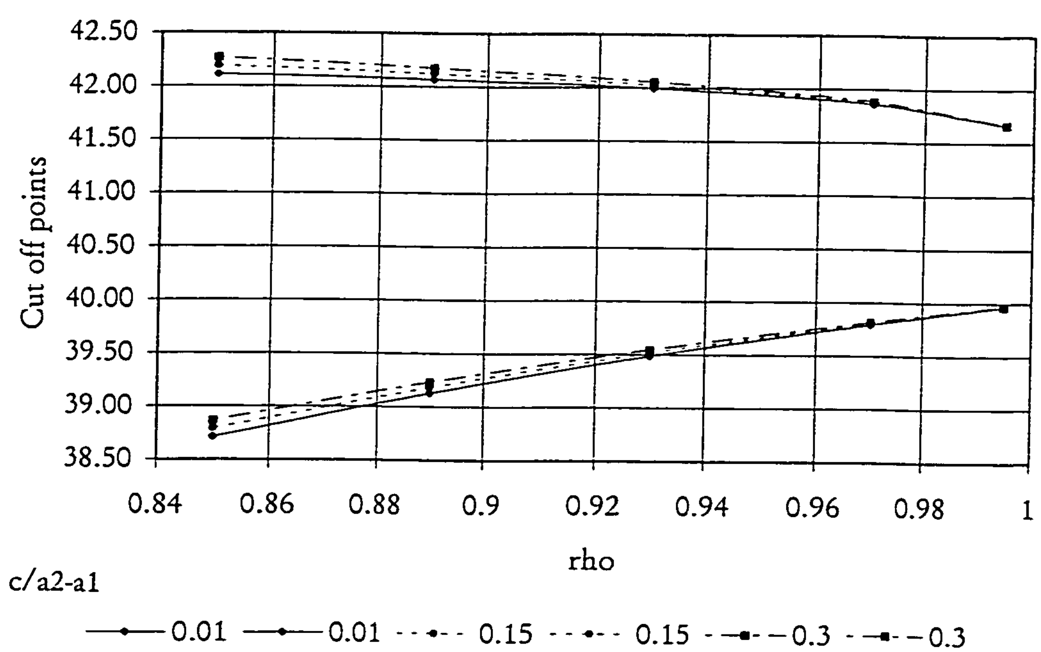


Figure 5-35: w1, w2 versus measurment error (rho) at
a2-r/a1-a2 = 6.5, sigma = 1.25
Special case II

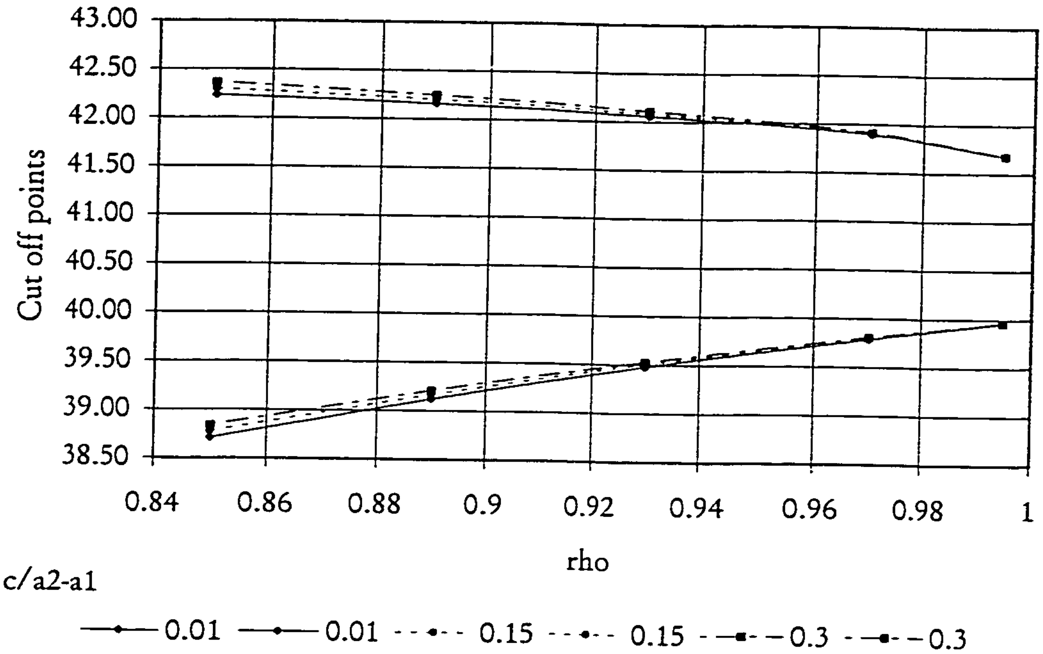


Figure 5-36: w1, w2 versus measurment error (rho) at
a2-r/a1-a2 = 8, sigma = 1.25
Special case II

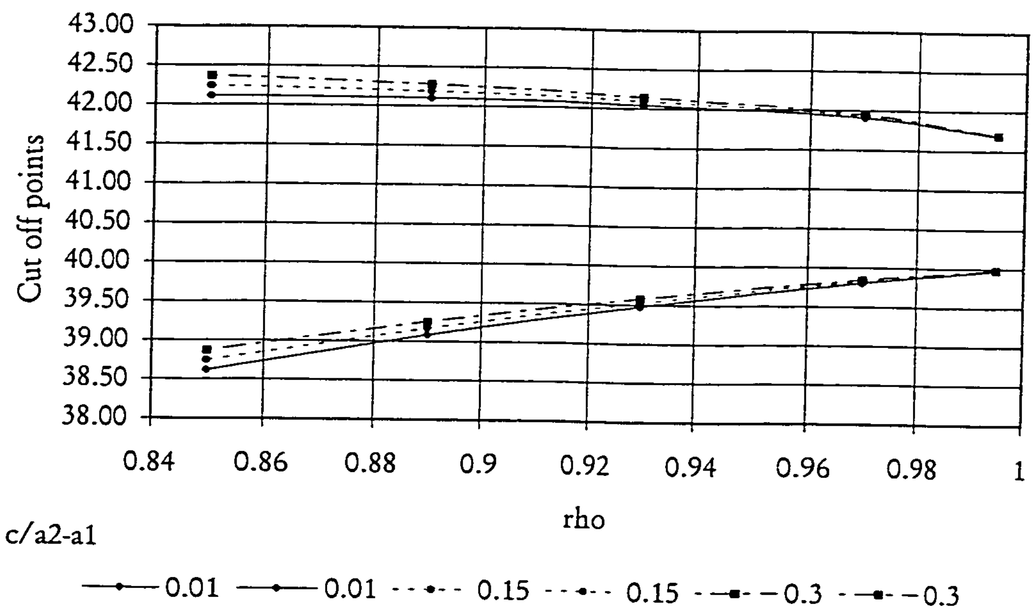


Figure 5-37: w1, w2 versus measurment error (rho) at
a2-r/a1-a2 = 4, sigma = 1.75
Special case II

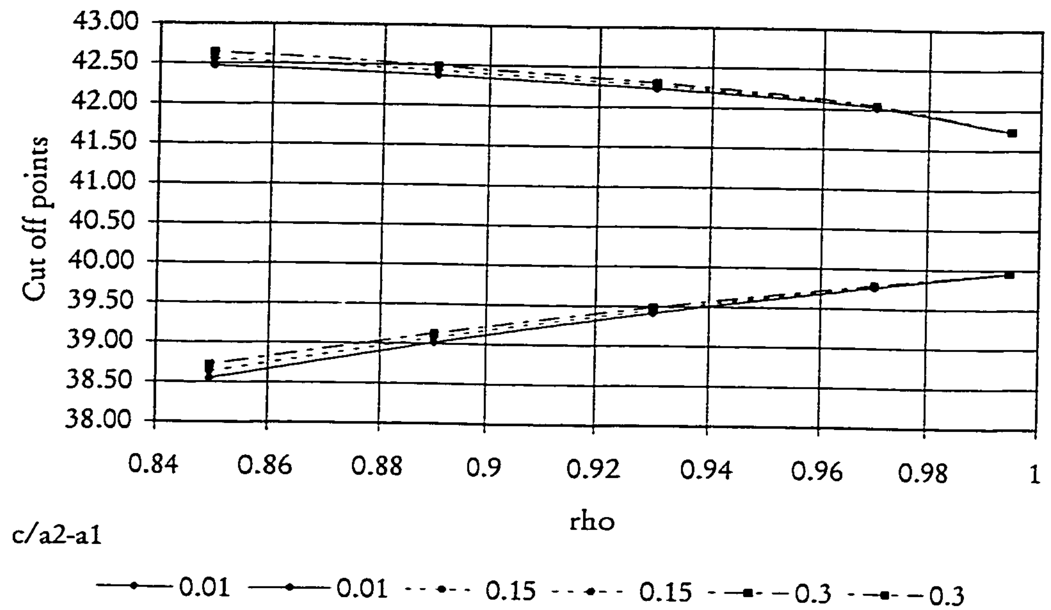


Figure 5-38: w1, w2 versus measurment error (rho) at
a2-r/a1-a2 = 6.5, sigma = 1.75
Special case II

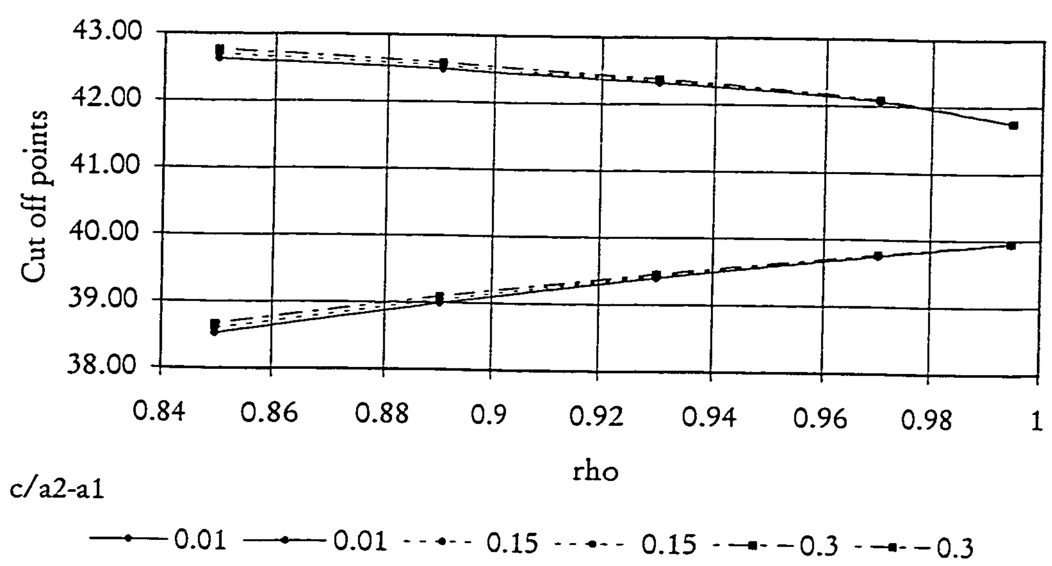


Figure 5-39: w1, w2 versus measurment error (rho) at
a2-r/a1-a2 = 8, sigma = 1.75
Special case II

3. The changes in cut off points with measurement error are higher at lower cost because of rapid changes in mean at lower costs (as shown in figures 5-34 to 5-39).
4. The variation in w_2 is higher than w_1. This is because the mean is far from w_2 and the effect of changing, w_2 is less as compared to w_1.

At higher level of costs, the results tend to remain conservative, (both cut off point tend to remain higher) i.e., it is better to lose+ profit than paying the penalty.

5.4 CONCLUSION

In this chapter, a process targeting model for multi-class screening has been developed for the case of an error prone measurement system. Sensitivity analysis is also performed to study the effect of different parameters. Despite the effect of target on the optimal settings, the model showed almost the same nullifying effect on the error by keeping the level of profit on approximately the same level as in model 2.

CHAPTER 6

MODEL COMPARISONS

6.1 INTRODUCTION

The purpose of this chapter is to compare the multi-class screening targeting models developed in this thesis. The comparison will be done analytically and numerically. In section 6.2, relationships between the developed models will be investigated. The verification of the generalized models are show by reduction of these to the simpler models .

In section 6.3, the numerical comparison of the models is presented. In this analysis, the effect of gain in profit is studied at different parameter levels, for the case where a more realistic model is used instead. In order to do that, the optimal settings from the basic models is substituted in the more general models in order to find out the 'actual expected profit' on which the system is operating. The expected gain in profit for the case where more general model is used is then estimated by finding the difference in 'actual expected profit' and the optimal expected profit obtained by the generalized model. The results are shown as a percentage gain in profit. Finally, in section 6.3 the conclusion of this chapter is presented.

6.2 ANALYTIC COMPARISONS

In this section, the Model 1 i.e., Min Koo Lee and Joon Soon Jang (1997) model along with the three models developed in this thesis work are compared analytically. In section 6.2.1, Model 1 will be derived from Model 2. In section 6.3.2, Model 1 will be derived from Model 3. The case of Model 2 versus model 4 will be presented in section 6.3.3. In the following section 6.3.4, Model 3 will be derived from Model 4.

6.2.1 MODEL 1 VS MODEL 2

In this section Model 1 will be derived from model 2. From chapter 2 and 3 Model 1 and 2 can be written as:

MODEL 1:

$$EPM1 = a_1[\Phi(-\Gamma_1)] + a_2\Phi(\Gamma_1) + (r - a_2)\Phi(\Gamma_2) - c_0 - c_i - c(L_1 - \Gamma_1\sigma_Y)$$

MODEL 2:

$$\begin{aligned} EPM2 = {} & a_1\Phi(-\delta_1) + a_2[\Phi(\delta_1) - \Phi(\delta_2)] + r\Phi(\delta_2) \\ & - b_{21}\int_{\delta_1}^{\infty}\int_{\Gamma_2}^{\Gamma_1}\psi(v,u)dvdu - b_{s1}\int_{\delta_1}^{\infty}\int_{-\infty}^{\Gamma_2}\psi(v,u)dvdu - b_{s2}\int_{\delta_2}^{\delta_1}\int_{-\infty}^{\Gamma_2}\psi(v,u)dvdu \\ & - c_0 - c_i - c(L_1 - \Gamma_1\sigma_Y) \end{aligned}$$

Where:

$$\Gamma_1 = \frac{L_1 - \mu}{\sigma_Y},\ \Gamma_2 = \frac{L_2 - \mu}{\sigma_Y},\ \delta_1 = \frac{w_1 - \mu}{\frac{\sigma_Y}{\rho}},\ \delta_2 = \frac{w_2 - \mu}{\frac{\sigma_Y}{\rho}}$$

$$\psi(v,u)=\frac{1}{2\pi\sqrt{1-\rho^2}}e^{\frac{1}{2(1-\rho^2)}\left\{(u)^2+(v)^2-2\rho uv\right\}} \tag{6-1}$$

Where:

$$v=\frac{Y-\mu}{\sigma_Y},\quad u=\frac{X-\mu}{\frac{\sigma_Y}{\rho}},$$

From section 3.2.2, the relationship between the observed value and the actual value was described to be as

$$X=Y+\varepsilon \tag{6-2}$$

Moreover, it was shown that $\sigma_X^2=\sigma_Y^2+\sigma_\varepsilon^2$. Where 'ε' is error in measurement and it is assumed to be unbiased and normally distributed i.e., $\varepsilon\sim N(0,\sigma_\varepsilon^2)$. Where from equation (3.2.2)

$$\rho=1-\frac{\sigma_\varepsilon^2}{\sigma_X^2}=\frac{\sigma_Y^2}{\sigma_X^2}=\frac{\sigma_\varepsilon^2}{\sigma_Y^2+\sigma_\varepsilon^2}$$

From the above relationship, it is clear that as $\sigma_\varepsilon\to 0$, $\rho\to 1$. And at $\rho=1$ it is clear that

$$\sigma_X^2=\sigma_Y^2$$

As $\sigma_\varepsilon=0$, and the error was assumed to unbiased i.e.,

$$E(\varepsilon)=0$$

Equation (6-2) can be rewritten as:

$$E(X)=E(Y)$$

And since X and Y are normal with the same mean and variance then we have the two random variables X and Y equal i.e.,

$$X = Y$$

Therefore at $\rho = 1$

$$\delta_1 = \Gamma_1 \ \& \ \delta_2 = \Gamma_2 \qquad (6\text{-}3)$$

Now as $\rho \rightarrow 1$, it is clear that equation (6-1) will vanish. It can be shown that at $\rho = 1$, by using L'hospital rule that numerator will reach zero quicker that the denominator. Therefore equation (6-1) can be written as

$$\psi(v, u) = 0 \qquad (6\text{-}4)$$

By equation (6-4) the error penalty term will vanish at $\rho = 1$ and the model can be re-written as:

$$EPM2 = a_1\Phi(-\delta_1) + a_2[\Phi(\delta_1) - \Phi(\delta_2)] + r\Phi(\delta_2) - c_0 - c_i - c(L_1 - \Gamma_1\sigma_Y) \qquad (6\text{-}5)$$

Using equation (6-3), equation (6-5) will be reduced to Model 1 i.e.,

$$EPM2 = a_1[\Phi(-\Gamma_1)] + a_2\Phi(\Gamma_1) + (r - a_2)\Phi(\Gamma_2) - c_0 - c_i - c(L_1 - \Gamma_1\sigma_Y)$$

Therefore at $\rho = 1$:

$$EPM2 = EPM1$$

So as $\rho \rightarrow 1$ $EPM2 \rightarrow EPM1$. Therefore at higher level of ρ the effect of error will be reduced and Model 1 can be used as an approximation of Model 2. This is also demonstrated empirically in Table 3-3, in which cut off points converge to the respective specification limits.

6.2.2 MODEL 1 VS MODEL 3

In this section, Model 1 will be derived from model 3. From chapter 2 and 4 Model 1 and 2 can be written as:

MODEL 1:

$$EPM1 = a_1[\Phi(-\Gamma_1)] + a_2\Phi(\Gamma_1) + (r - a_2)\Phi(\Gamma_2) - c_0 - c_i - c(L_1 - \Gamma_1\sigma_y)$$

MODEL 3:

$$\begin{aligned} EPM3 &= EPM1 \\ &\quad - K[\varphi(\Gamma_2)[\sigma^2 z\varphi(\Gamma_2) + 2\sigma(\mu - t)] + [\sigma_Y^2 + (\mu - t)^2][1 - \Phi(\Gamma_2)]] \end{aligned}$$

The reduction of Model 3 into Model 1 is trivial. i.e., at K = 0, Model 3 will be reduced to Model 1. or

$$EPM3 = EPM1$$

It is clear that as $K \to 0$ $EPM3 \to EPM1$. Therefore, as K approaches zero, Model 1 can be used as a good approximation for Model 3. However, there is one important point, by comparing special cases discussed in chapter 4, i.e.,

SPECIAL CASE I:

$$\begin{aligned} EPM3 &= EPM1 \\ &\quad - K[\varphi(\Gamma_2)[\sigma^2 z\varphi(\Gamma_2)] + \sigma_Y^2[1 - \Phi(\Gamma_2)]] \end{aligned}$$

SPECIAL CASE II:

$$\begin{aligned} EPM3 &= EPM1 \\ &\quad - K[\varphi(\Gamma_2)[\sigma^2 z\varphi(\Gamma_2) + 2\sigma(\mu - L_1)] + [\sigma_Y^2 + (\mu - L_1)^2][1 - \Phi(\Gamma_2)]] \end{aligned}$$

it is clear that the penalty terms in special case II can vanish slowly as compared to special case I. Therefore as $K \to 0$ in special case II EPM3 will tend to slowly converge to EPM1. Hence care must required in the approximation when the special case II is being used i.e., the target being set at L_1.

6.2.3 MODEL 2 VS MODEL 4

In this section, Model 2 will be derived from model 4. From chapter 5 and 3 Model 2 and 4 can be written as:

MODEL 1:

$$\begin{aligned}EPM2 &= a_1\Phi(-\delta_1) + a_2[\Phi(\delta_1) - \Phi(\delta_2)] + r\Phi(\delta_2)\\ &\quad - b_{21}\int_{\delta_1}^{\infty}\int_{\Gamma_2}^{\Gamma_1}\psi(v,u)dvdu - b_{s1}\int_{\delta_1}^{\infty}\int_{-\infty}^{\Gamma_2}\psi(v,u)dvdu - b_{s2}\int_{\delta_2}^{\delta_1}\int_{-\infty}^{\Gamma_2}\psi(v,u)dvdu\\ &\quad - c_0 - c_i - c(L_1 - \Gamma_1\sigma_Y)\end{aligned}$$

MODEL 4:

$$\begin{aligned}EPM4 &= EPM2\\ &\quad - K\left[\varphi(\Gamma_2)[\sigma^2 z\varphi(\Gamma_2) + 2\sigma(\mu - t)] + [\sigma_Y^2 + (\mu - t)^2][1 - \Phi(\Gamma_2)]\right]\end{aligned}$$

The model reduction is similar to as in section 6.2.2. i.e., at K = 0

$$EPM4 = EPM2$$

It is clear that as $K \to 0$ $EPM4 \to EPM2$. Therefore, as K approaches zero Model 2 can be used as a good approximation for Model 4. As with model 3,the important point is, by comparing special cases discussed in chapter 4, i.e.,

SPECIAL CASE I:

$$\begin{aligned}EPM4 &= EPM2\\ &\quad - K\left[\varphi(\Gamma_2)[\sigma^2 z\varphi(\Gamma_2)] + \sigma_Y^2[1 - \Phi(\Gamma_2)]\right]\end{aligned}$$

SPECIAL CASE II:

$$\begin{aligned}EPM4 &= EPM2\\ &\quad - K\left[\varphi(\Gamma_2)[\sigma^2 z\varphi(\Gamma_2) + 2\sigma(\mu - L_1)] + [\sigma_Y^2 + (\mu - L_1)^2][1 - \Phi(\Gamma_2)]\right]\end{aligned}$$

it is clear that the penalty terms in special case II will vanish slowly as compared to special case I. Therefore, care must required in the approximation of EPM2 as EPM4 when the special case II is being used i.e., the limit being set at L_1.

6.2.4 MODEL 3 VS MODEL 4

In this section, Model 3 will be derived from model 4. From chapter 2 and 3 Model 1 and 2 can be written as:

MODEL 3:

$$\begin{aligned} EPM3 = {} & EPM1 \\ & - K\left[\varphi(\Gamma_2)\left[\sigma^2 z\varphi(\Gamma_2) + 2\sigma(\mu - t)\right] + \left[\sigma_Y^2 + (\mu - t)^2\right]\left[1 - \Phi(\Gamma_2)\right]\right] \end{aligned}$$

MODEL 4:

$$\begin{aligned} EPM4 = {} & EPM2 \\ & - K\left[\varphi(\Gamma_2)\left[\sigma^2 z\varphi(\Gamma_2) + 2\sigma(\mu - t)\right] + \left[\sigma_Y^2 + (\mu - t)^2\right]\left[1 - \Phi(\Gamma_2)\right]\right] \end{aligned}$$

It is clear form the above equations that the last non-uniformity terms of both the models are similar. Also from section 6.2.1 it is shown that EPM2 = EPM1 at $\rho = 1$; Therefore at $\rho = 1$

$$\begin{aligned} EPM4 = {} & EPM1 \\ & - K\left[\varphi(\Gamma_2)\left[\sigma^2 z\varphi(\Gamma_2) + 2\sigma(\mu - t)\right] + \left[\sigma_Y^2 + (\mu - t)^2\right]\left[1 - \Phi(\Gamma_2)\right]\right] \end{aligned}$$

or

$$EPM4 = EPM3$$

As EPM4 can be reduced to EPM2 at $\rho = 1$, and EPM3 can be reduced to EPM1 at $K = 0$, the proof for EPM4 = EPM1 at $\rho = 1$ and $K = 0$ is trivial.

6.3 NUMERICAL COMPARISONS

In this section, the sensitivity analysis is presented for the analytical comparisons presented in section 6.2.

In section 6.3.1, results from the analysis of Model 1 versus Model 2 are presented. In section 6.3.2, Model 1 and Model 3 will be compared. The case of Model 2 versus model 4 will be presented in section 6.3.3. In section 6.3.4, Model 3 will be compared with Model 4.

6.3.1 MODEL 1 VS MODEL 2

In this section, model 1 and model 2 will be compared. In order to find the percentage gain if measurement error is present and Model 1 is used instead of Model 2, the optimal mean obtain by Model 1 is substituted in Model 2 and the cut off values are taken at specification limits i.e.,

$$w_1=L_1,\ w_2=L_2$$

The result, in Table 6-1, shows the %gain in profit if Model 2 is used, instead of Model 1, in order to find out the optimal process parameter settings. The parameter levels are taken as in chapter 2 i.e., the parameters varied are $\frac{c}{a_1-a_2}, \frac{a_2-r}{a_1-a_2}$ & σ_y, where cost is represented by $\frac{c}{a_1-a_2}$, another dimensionless parameter $\frac{a_2-r}{a_1-a_2}$ represents the selling price while the error is represented by ρ. The results are shown graphically in figures 6-1 to 6-4.

The results show significant % gain in profit with increasing cost i.e., if the cost of production is higher, there will be significant gains in profit if more general model i.e., 'Model 2' is used. The gain in profit is also high at higher levels of measurement errors showing the effectiveness of Model 2 with increasing measurement error.

TABLE 6-1: Expected % gain in profit at different parameter settings. Model 1 Vs Model 2

σ_v	$\frac{a_2 - r}{a_1 - a_1}$	ρ	$\frac{c}{a_1 - a_2}$ 0.01	0.15	0.3
1.25	4	0.85	0.161	6.147	4.922
		0.89	0.112	4.764	4.048
		0.93	0.075	3.508	3.147
		0.97	0.044	2.200	2.070
		0.995	0.019	0.927	0.896
	6	0.85	0.140	2.867	135.524
		0.89	0.103	2.310	176.829
		0.93	0.075	1.783	389.855
		0.97	0.048	1.185	189.801
		0.995	0.023	0.525	27.486
	8	0.85	0.133	2.351	15.138
		0.89	0.100	1.916	12.116
		0.93	0.074	1.499	9.198
		0.97	0.050	1.012	5.879
		0.995	0.024	0.454	2.442
1.75	4	0.85	0.217	7.901	5.176
		0.89	0.147	6.000	4.240
		0.93	0.095	4.299	3.262
		0.97	0.054	2.582	2.090
		0.995	0.023	1.041	0.877
	6	0.85	0.180	3.228	62.303
		0.89	0.127	2.542	61.844
		0.93	0.087	1.908	62.682
		0.97	0.054	1.221	68.663
		0.995	0.025	0.524	137.834
	8	0.85	0.165	2.537	17.932
		0.89	0.118	2.016	13.998
		0.93	0.083	1.532	10.313
		0.97	0.053	0.997	6.325
		0.995	0.025	0.435	2.531

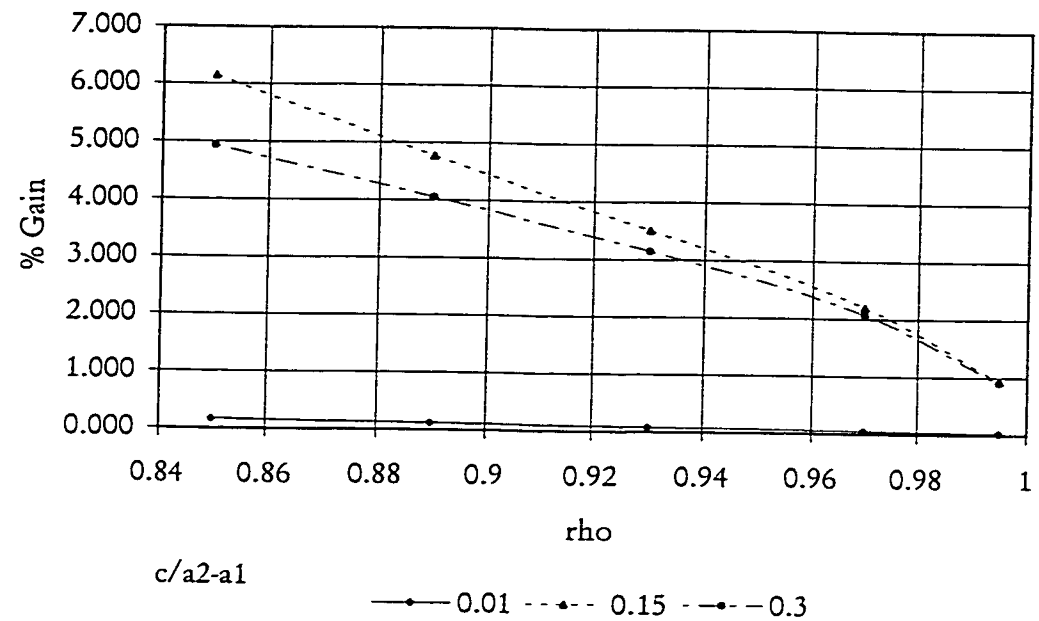


Figure 6-1: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 4, sigma = 1.25

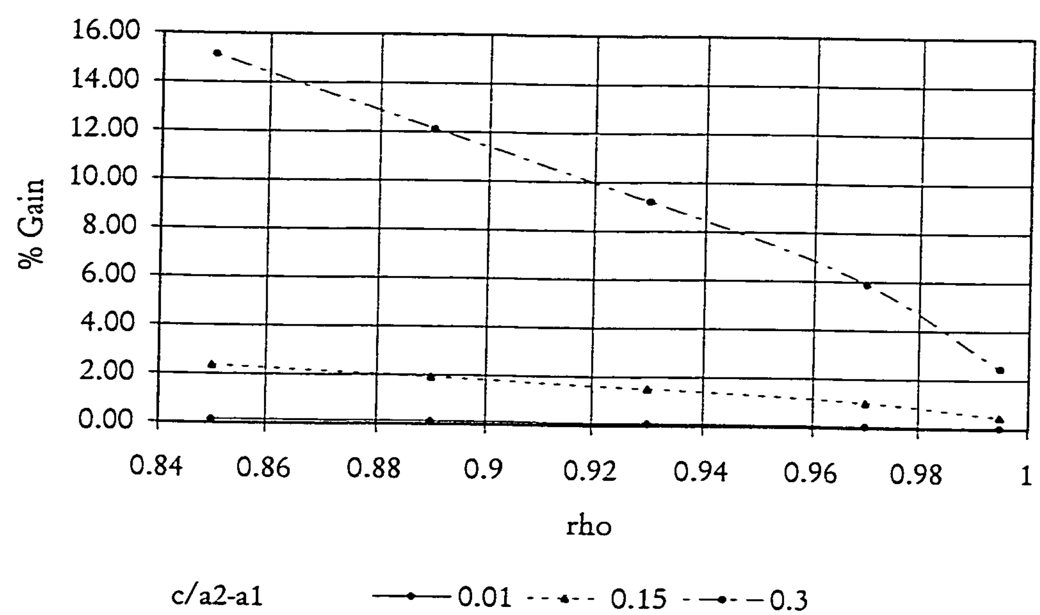


Figure 6-2: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 8, sigma = 1.25

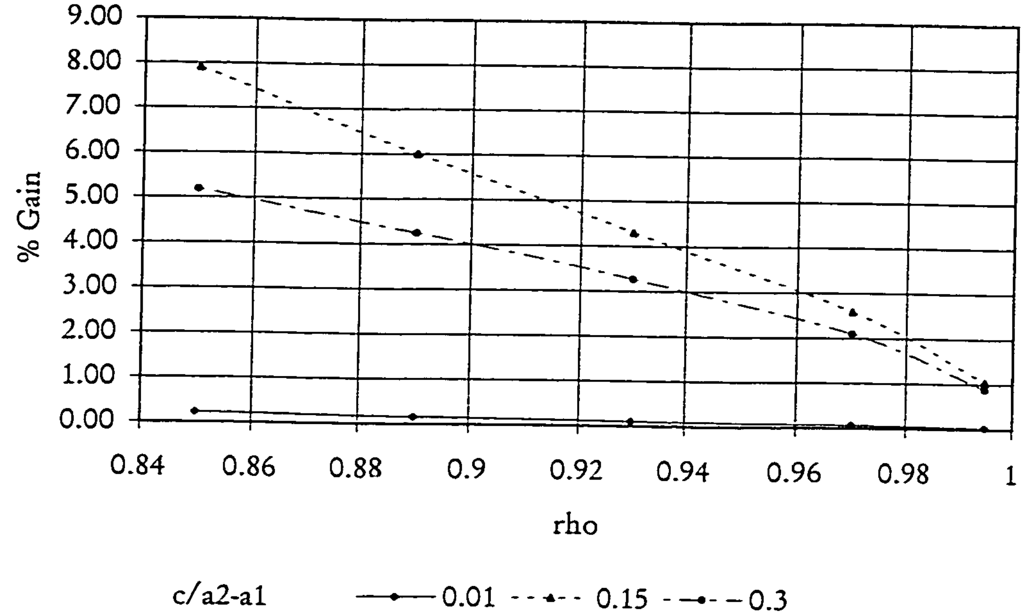


Figure 6-3: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 4, sigma = 1.75

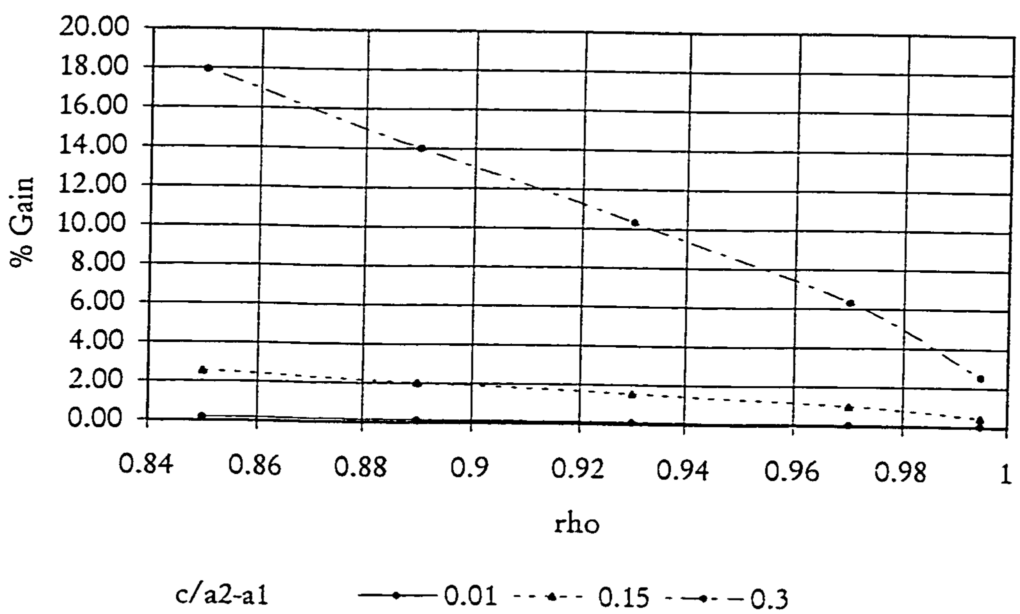


Figure 6-4: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 8, sigma = 1.75

The results are summarized as follows:

1. The % gain increases with the increase in measurement error. This result is expected. As discussed in 6.2.1 if the error is very low or $\rho \rightarrow 1$ Model 2 can be approximated by Model 1. However, as the error goes up the numerical results show significant gains.
2. The % gain increases with the increase in the cost. From the sensitivity analysis of chapter 3, it was clear that as the cost goes up, mean comes down, closer to the specification limits, in turn increasing the effect of error by increasing the chances of misclassification. As the effect of error increases, the %gain increases with the use of Model 2.

6.3.2 MODEL 1 VS MODEL 3

In this section, model 1 and model 3 will be compared. In order to find out the percentage gain, if uniformity penalty is present and Model 3 is used instead of Model 1, optimal mean obtain by Model 1 is substituted in Model 3. The analysis was performed for both the special cases. The result for special case I (Mean as target) is presented in Table 6-2. While for special case II the results are shown in Table 6-3. The value of K is taken at 0.05 in order to see the effect of K close to zero. The parameters taken are same as in chapter 4 i.e., $\frac{c}{a_1 - a_2}, \frac{a_2 - r}{a_1 - a_2}$ & σ_y, and the levels are taken to be the same. It is evident from comparison of Table 6-2 and 6-3 that the % gain for special case I is negligible cross more than 1% at very few combinations. While in case of special case II the gains are significant. The results are shown in figures 6-5 to 6-12

TABLE 6-2: Expected % gain in profit at different parameter settings. Model 1 Vs Model 3 (Special case I)

σ_v	$\frac{a_2 - r}{a_1 - a_1}$	$\frac{c}{a_1 - a_2}$ 0.01	0.05	0.1	0.15	0.2	0.25	0.3
	4.0	0.0003	0.0031	0.0105	0.0329	1.3448	0.0492	0.0286
	5.0	0.0003	0.0030	0.0090	0.0213	0.0636	0.2587	0.0548
	5.5	0.0003	0.0030	0.0085	0.0185	0.0442	0.3293	0.0889
1.5	6.0	0.0004	0.0030	0.0081	0.0169	0.0358	0.1195	0.1713
	6.5	0.0004	0.0029	0.0078	0.0154	0.0299	0.0721	3.6803
	7.0	0.0004	0.0029	0.0075	0.0142	0.0255	0.0511	0.1824
	7.5	0.0004	0.0029	0.0073	0.0134	0.0232	0.0425	0.1059
	8.0	0.0004	0.0029	0.0070	0.0127	0.0212	0.0363	0.0740
	4.0	0.0013	0.0102	0.0301	0.0886	1.0398	0.1022	0.0581
	5.0	0.0014	0.0097	0.0252	0.0554	0.1646	0.4292	0.1093
	5.5	0.0015	0.0095	0.0236	0.0478	0.1108	1.2906	0.1715
1.75	6.0	0.0015	0.0094	0.0224	0.0432	0.0884	0.3158	0.3043
	6.5	0.0015	0.0092	0.0213	0.0393	0.0731	0.1777	1.4578
	7.0	0.0015	0.0090	0.0203	0.0359	0.0619	0.1224	0.5040
	7.5	0.0015	0.0089	0.0196	0.0338	0.0560	0.1008	0.2624
	8.0	0.0016	0.0088	0.0190	0.0319	0.0510	0.0853	0.1759
	4.0	0.0043	0.0258	0.0689	0.1964	1.2026	0.1799	0.1008
	5.0	0.0044	0.0242	0.0569	0.1194	0.3633	0.6433	0.1876
	5.5	0.0045	0.0235	0.0530	0.1024	0.2355	11.308	0.2868
2	6.0	0.0045	0.0230	0.0502	0.0922	0.1852	0.7438	0.4782
	6.5	0.0046	0.0225	0.0476	0.0835	0.1518	0.3806	1.4713
	7.0	0.0046	0.0220	0.0451	0.0760	0.1279	0.2535	1.3239
	7.5	0.0046	0.0216	0.0435	0.0715	0.1153	0.2063	0.5782
	8.0	0.0046	0.0212	0.0420	0.0674	0.1047	0.1733	0.3674
	4.0	0.0110	0.0553	0.1368	0.3851	1.4715	0.2831	0.1570
	5.0	0.0113	0.0514	0.1122	0.2277	0.7270	0.8995	0.2906
	5.5	0.0113	0.0498	0.1041	0.1943	0.4502	7.1567	0.4351
2.25	6.0	0.0114	0.0486	0.0985	0.1744	0.3484	1.6787	0.6916
	6.5	0.0114	0.0473	0.0932	0.1577	0.2829	0.7461	1.6831
	7.0	0.0115	0.0460	0.0883	0.1434	0.2370	0.4767	3.8240
	7.5	0.0115	0.0451	0.0850	0.1349	0.2133	0.3827	1.1952
	8.0	0.0115	0.0442	0.0819	0.1270	0.1933	0.3186	0.7053

TABLE 6-3: Expected % gain in profit at different parameter settings. Model 1 Vs Model 3 (Special case II)

σ_v	$\frac{a_2 - r}{a_1 - a_1}$	$\frac{c}{a_1 - a_2}$ 0.01	0.05	0.1	0.15	0.2	0.25	0.3
	4.0	11.32	6.64	6.15	9.09	49.33	4.04	1.68
	5.0	11.56	6.66	5.68	6.41	11.96	19.25	3.79
	5.5	11.67	6.70	5.56	5.87	8.53	83.24	6.44
1.5	6.0	11.77	6.74	5.50	5.56	7.17	18.30	12.08
	6.5	11.87	6.79	5.46	5.33	6.29	10.79	55.99
	7.0	11.98	6.85	5.44	5.15	5.68	7.93	25.25
	7.5	12.06	6.90	5.44	5.06	5.39	6.86	13.55
	8.0	12.15	6.96	5.44	4.99	5.15	6.12	9.55
	4.0	16.01	9.70	9.34	14.87	42.30	5.66	2.46
	5.0	16.37	9.75	8.59	10.11	21.32	22.05	5.39
	5.5	16.54	9.81	8.41	9.20	14.35	389.37	8.80
1.75	6.0	16.69	9.88	8.32	8.69	11.81	38.61	15.20
	6.5	16.84	9.96	8.25	8.31	10.21	19.50	43.53
	7.0	17.02	10.06	8.23	8.02	9.15	13.59	67.12
	7.5	17.15	10.15	8.23	7.87	8.63	11.53	26.72
	8.0	17.29	10.24	8.24	7.75	8.22	10.15	17.30
	4.0	21.86	13.65	13.61	23.50	41.79	7.52	3.40
	5.0	22.38	13.73	12.45	15.27	37.43	25.26	7.24
	5.5	22.63	13.82	12.17	13.77	23.23	108.69	11.41
2	6.0	22.85	13.92	12.03	12.95	18.58	90.83	18.52
	6.5	23.08	14.04	11.93	12.33	15.79	34.44	41.90
	7.0	23.35	14.20	11.88	11.86	13.98	22.28	359.51
	7.5	23.54	14.32	11.88	11.62	13.10	18.43	53.62
	8.0	23.75	14.46	11.90	11.43	12.42	15.96	30.28
	4.0	28.98	18.58	19.15	36.26	43.52	9.58	4.48
	5.0	29.71	18.69	17.40	22.24	66.51	28.77	9.28
	5.5	30.07	18.82	16.98	19.84	36.63	82.45	14.22
2.25	6.0	30.37	18.95	16.75	18.55	28.24	341.75	22.02
	6.5	30.71	19.13	16.60	17.58	23.49	61.10	43.22
	7.0	31.08	19.34	16.52	16.84	20.49	35.45	293.40
	7.5	31.36	19.51	16.50	16.46	19.08	28.39	119.68
	8.0	31.66	19.71	16.52	16.17	17.98	24.07	52.56

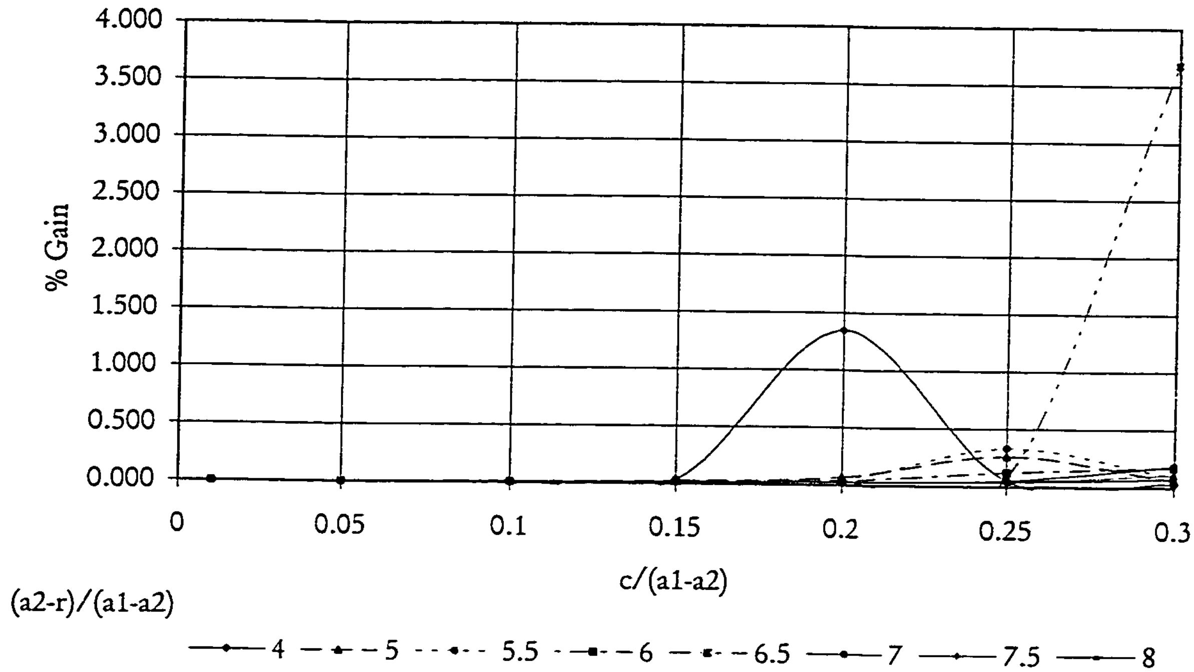


Figure 6-5: % Gain in Expected profit/a1, sigma = 1.5, k = 0.05, L2=40, L1=41.5
Model 3-1 vs Min & lee

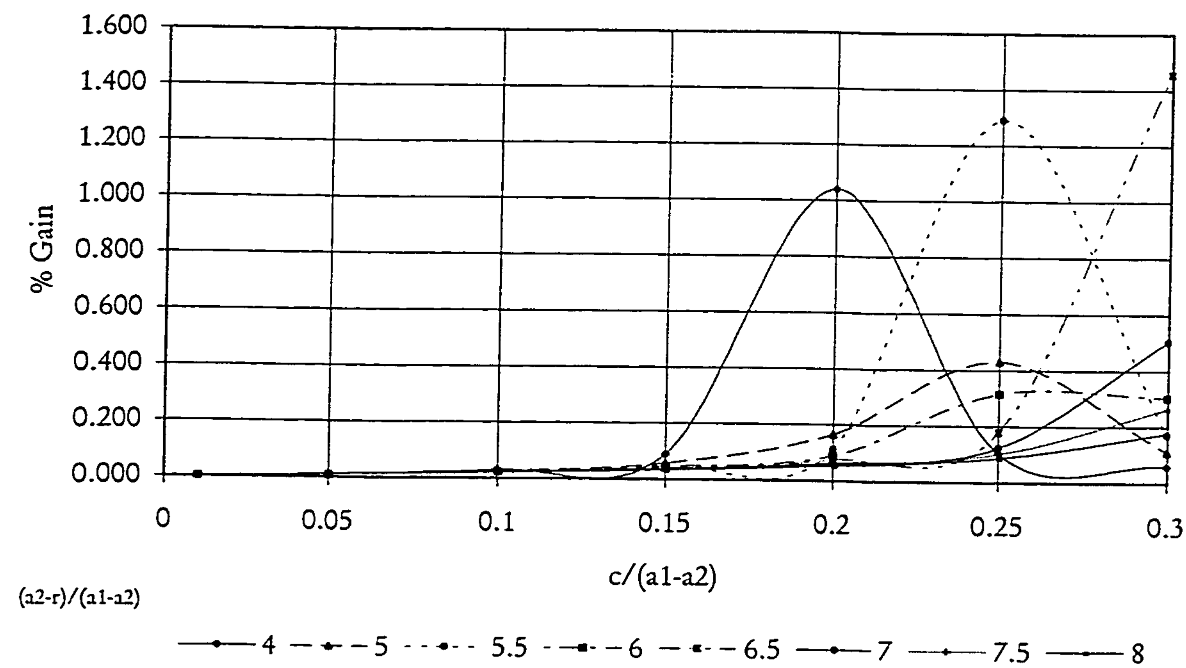


Figure 6-6: % Gain in Expected profit/a1, sigma = 1.75, k = 0.05, L2=40, L1=41.5
Model 3-1 vs Min & lee

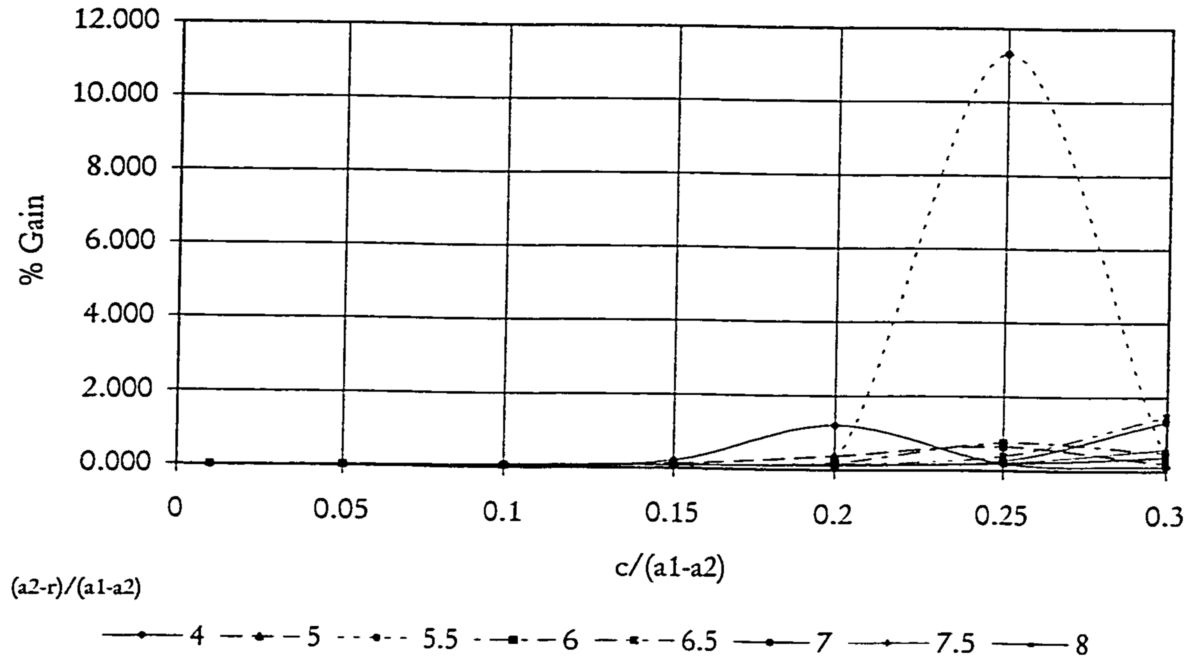


Figure 6-7: % Gain in Expected profit/a1, sigma = 2, k = 0.05, L2=40, L1=41.5
Model 3-1 vs Min & lee

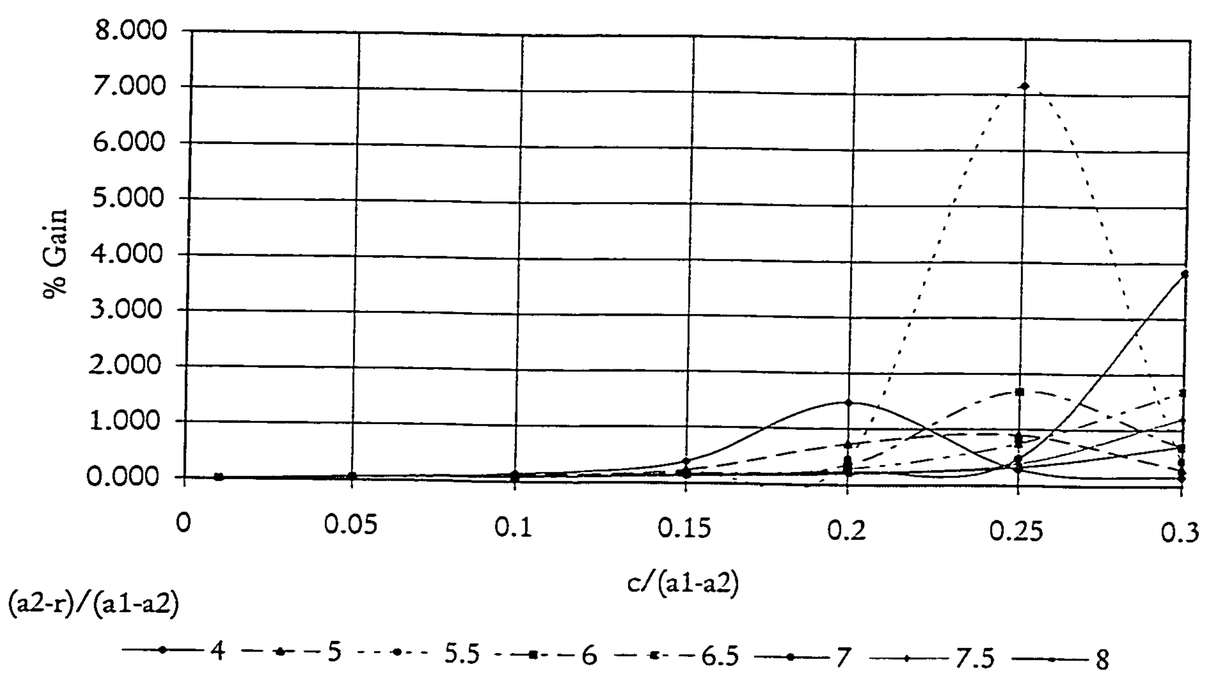


Figure 6-8: % Gain in Expected profit/a1, sigma = 2.25, k = 0.05, L2=40, L1=41.5
Model 3-1 vs Min & lee

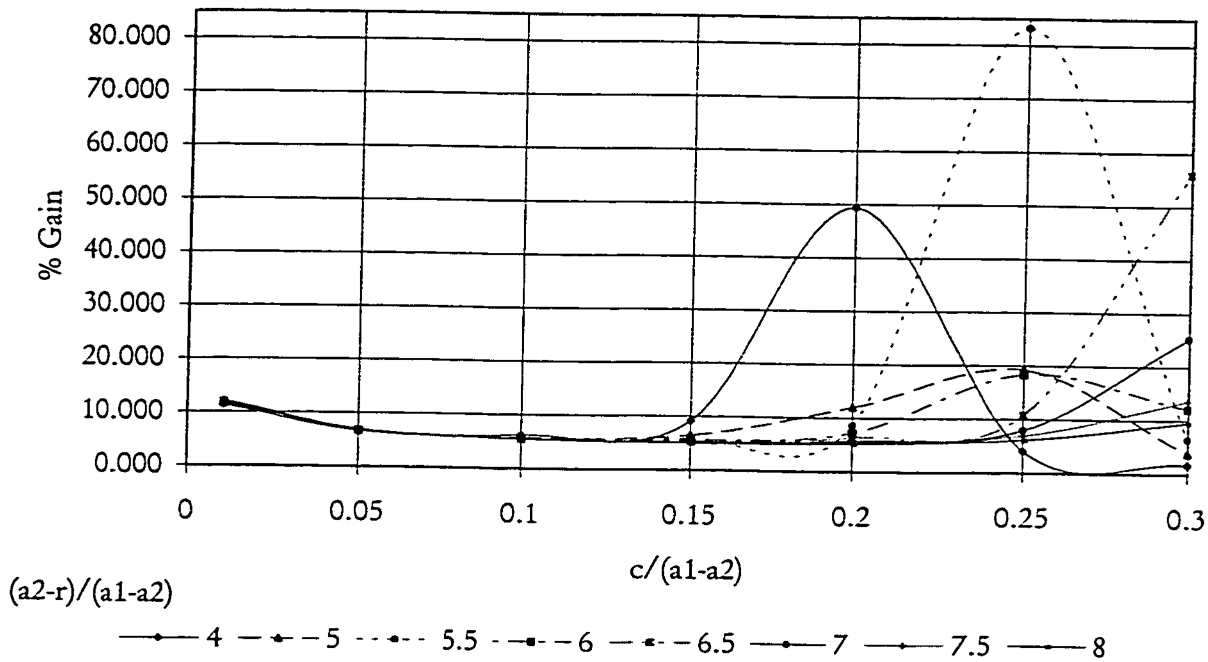


Figure 6-9 % Gain in Expected profit/a1, sigma = 1.5, k = 0.05, L2=40, L1=41.5
Model 3-2 vs Min & lee

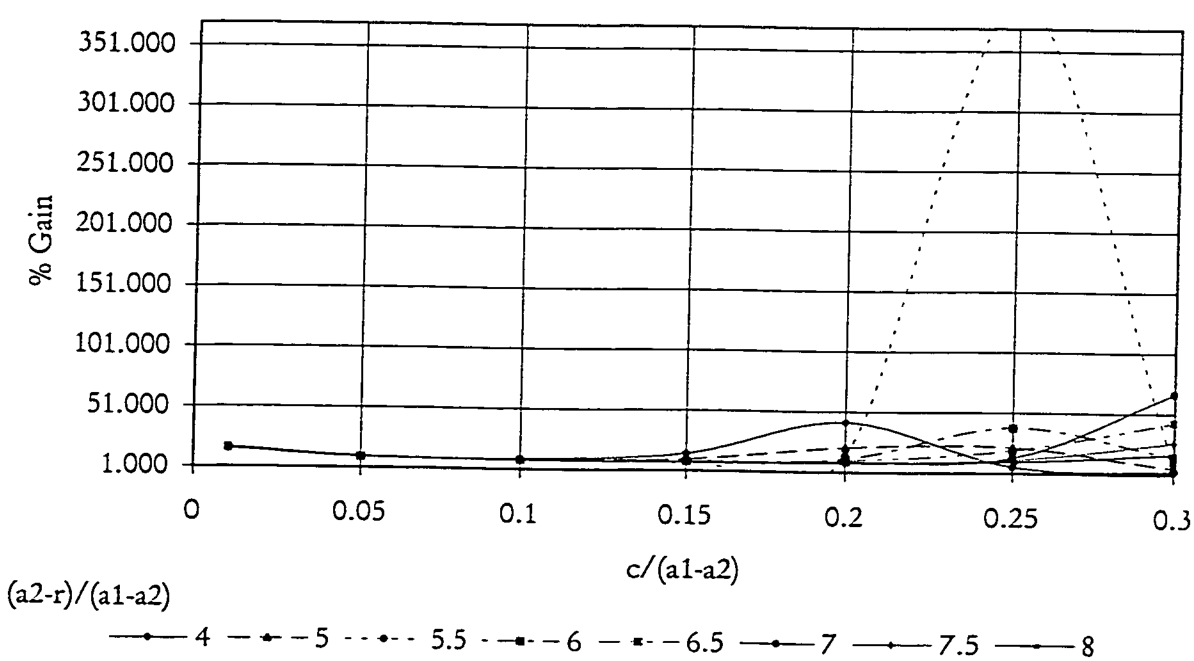


Figure 6-10: % Gain in expected profit/a1, sigma = 1.75, k = 0.05, L2=40, L1=41.5
Model 3-2 vs Min & lee

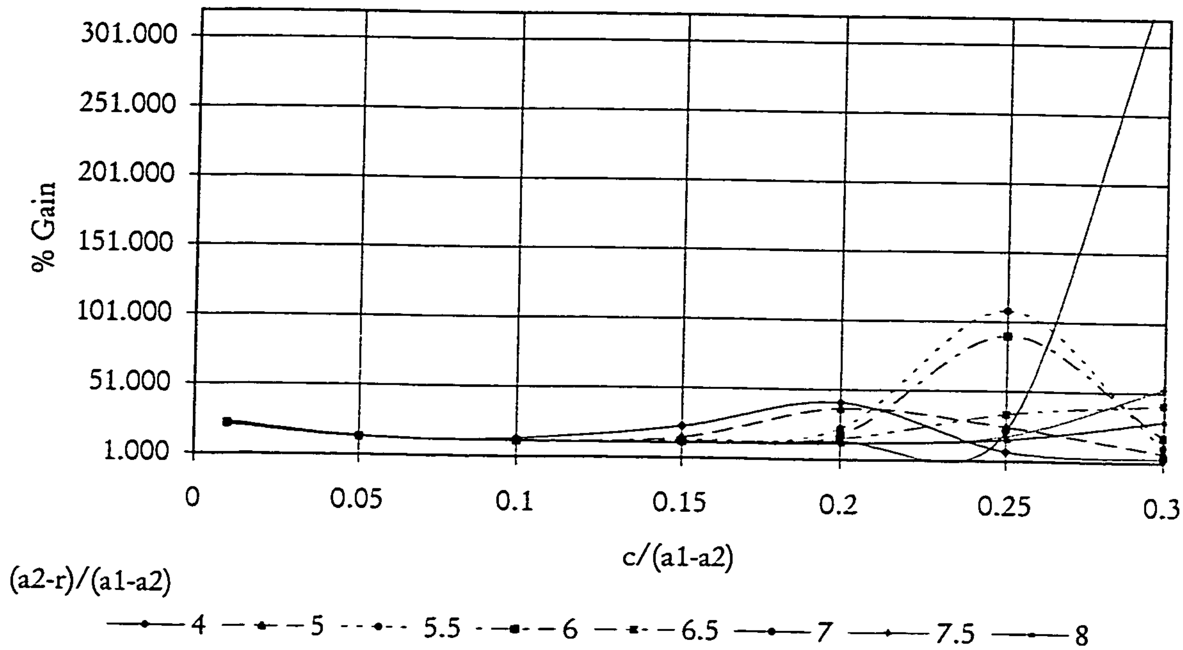


Figure 6-11: % Gain in Expected profit/a1, sigma = 2, k = 0.05, L2=40, L1=41.5
Model 3-2 vs Min & lee

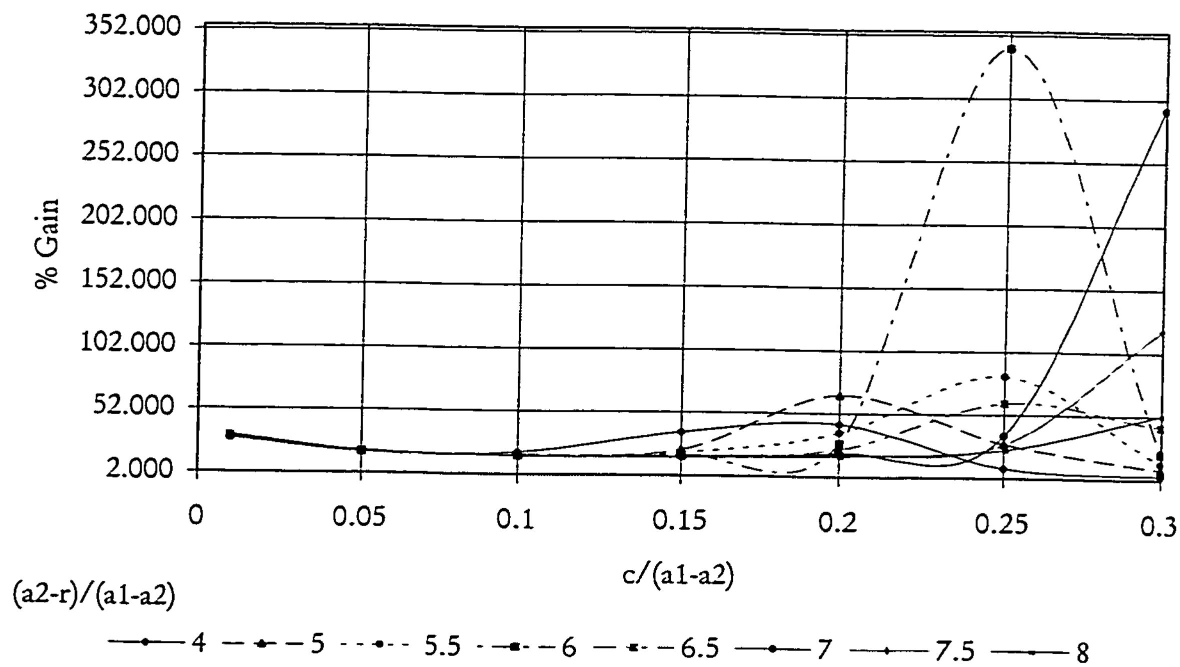


Figure 6-12: % Gain in Expected profit/a1, sigma = 2.25, k = 0.05, L2=40, L1=41.5
Model 3-2 vs Min & lee

The results can be summarized as follows:

1. The effect of non-uniformity penalty is more pronounced in special case II. The reason behind it is that the non-uniformity penalty term vanishes slowly in case II than I. This phenomena is discussed in section 6.2.2.
2. The % gain increases is more pronounced at higher level of costs.

6.3.3 MODEL 2 VS MODEL 4

In this section, model 2 and model 4 will be compared. In This comparison the effect on non-uniformity will discussed. In order to find the percentage gain if uniformity penalty is present and Model 4 is used instead of Model 2, the optimal mean obtained by Model 2 is substituted in Model 4. Also the cut off values are substituted in Model 4 obtained from Model 2.

The analysis was performed for both special cases. The result for special case I (Mean as target) is presented in Table 6-4. The results for special case II are shown in Table 6-5. The value of K is taken at 0.05 in order to see the effect of K close to zero. The parameters taken are same as in chapter 3 and 5 i.e., $\frac{c}{a_1 - a_2}, \frac{a_2 - r}{a_1 - a_2}$ & σ_y. The results are shown graphically in figures 6-13 to 6-16 for special case I and from figures 6-17 to 6-20 for special case II.

It is evident by comparing Table 6-4 and 6-5 that the % gain for special case I is small i.e., hardly reaching more than 0.5 – 1% at some levels. While with special case II, the gains are significant and the %gain is above 40% mark in a number of combinations. As a result, even at very low levels of K, for the case where the tar

TABLE 6-4: Expected % gain in profit at different parameter settings. Model 2 Vs Model 4 (Special case I)

σ_V	$\frac{a_2 - r}{a_1 - a_1}$	ρ	$\frac{c}{a_1 - a_2}$ 0.01	0.15	0.3
1.25	4	0.85	0.00000	0.00047	0.00148
		0.89	0.00000	0.00074	0.00206
		0.93	0.00000	0.00125	0.00300
		0.97	0.00000	0.00251	0.00490
		0.995	0.00001	0.00528	0.00798
	6	0.85	0.00000	0.00019	0.07883
		0.89	0.00000	0.00032	0.16050
		0.93	0.00000	0.00057	0.53098
		0.97	0.00000	0.00121	1.05412
		0.995	0.00002	0.00264	0.46759
	8	0.85	0.00000	0.00015	0.00351
		0.89	0.00000	0.00026	0.00495
		0.93	0.00000	0.00047	0.00722
		0.97	0.00000	0.00101	0.01137
		0.995	0.00002	0.00223	0.01832
1.75	4	0.85	0.00003	0.01408	0.01809
		0.89	0.00005	0.01866	0.02192
		0.93	0.00008	0.02609	0.02744
		0.97	0.00019	0.04072	0.03674
		0.995	0.00059	0.06548	0.04915
	6	0.85	0.00003	0.00540	0.19325
		0.89	0.00004	0.00735	0.24928
		0.93	0.00007	0.01051	0.34220
		0.97	0.00019	0.01684	0.53971
		0.995	0.00063	0.02807	0.94503
	8	0.85	0.00002	0.00421	0.05729
		0.89	0.00004	0.00579	0.06782
		0.93	0.00007	0.00832	0.08292
		0.97	0.00019	0.01341	0.10869
		0.995	0.00065	0.02257	0.14592

TABLE 6-5: Expected % gain in profit at different parameter settings. Model 2 Vs Model 4 (Special case II)

			$\frac{c}{a_1 - a_2}$		
σ_y	$\frac{a_2 - r}{a_1 - a_1}$	ρ	0.01	0.15	0.3
1.25	4	0.85	10.82357	12.85670	2.49945
		0.89	10.64364	11.70798	2.24994
		0.93	10.27781	10.29065	1.97092
		0.97	9.54702	8.42814	1.62926
		0.995	8.47460	6.52251	1.28347
	6	0.85	12.14880	7.38487	52.20975
		0.89	11.83807	6.71504	53.08808
		0.93	11.29780	5.91404	56.14462
		0.97	10.32270	4.89376	67.02767
		0.995	9.00007	3.87074	114.2240
	8	0.85	12.78219	6.88854	14.75395
		0.89	12.39890	6.23711	12.62678
		0.93	11.76799	5.47675	10.51868
		0.97	10.67301	4.53184	8.22009
		0.995	9.23695	3.60340	6.16830
1.75	4	0.85	23.24776	36.28450	4.64378
		0.89	22.66305	32.61009	4.31464
		0.93	21.69718	28.41657	3.93119
		0.97	20.00867	23.19844	3.41934
		0.995	17.73888	18.02779	2.85014
	6	0.85	25.44629	16.49124	41.50459
		0.89	24.59489	15.14311	40.78817
		0.93	23.31682	13.60736	40.38554
		0.97	21.25950	11.66110	40.51425
		0.995	18.69459	9.63071	41.69561
	8	0.85	26.47154	14.83418	47.27459
		0.89	25.49144	13.63864	41.17132
		0.93	24.06879	12.30102	34.91875
		0.97	21.85043	10.62994	27.78699
		0.995	19.17447	8.89310	21.17568

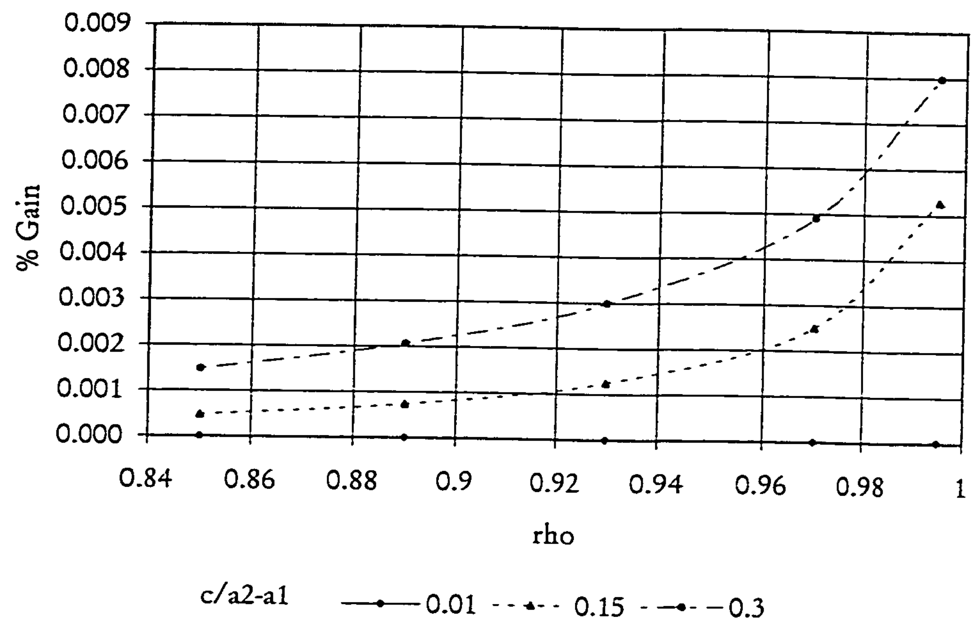


Figure 6-13: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 4, sigma = 1.25

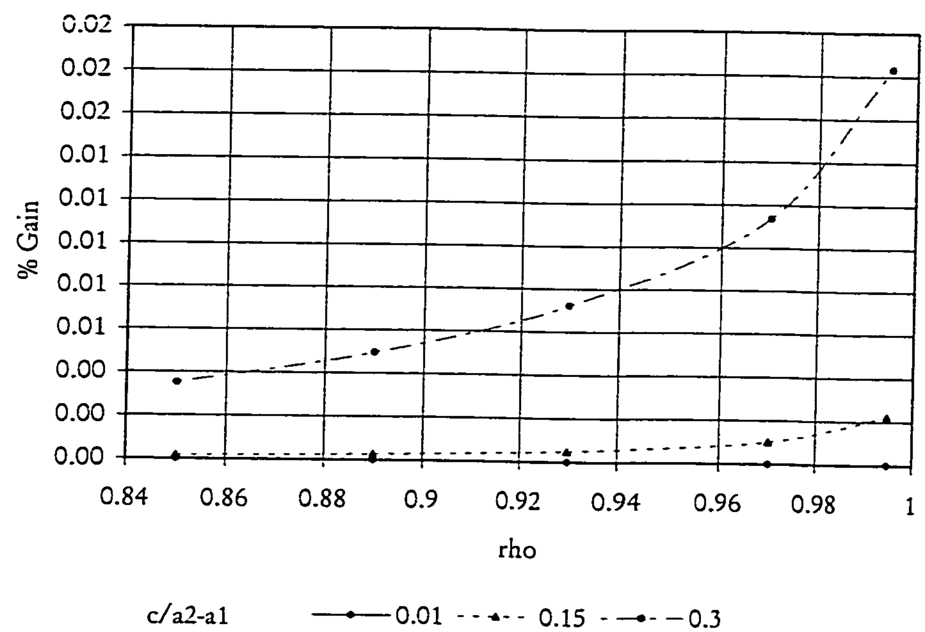


Figure 6-14: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 8, sigma = 1.25

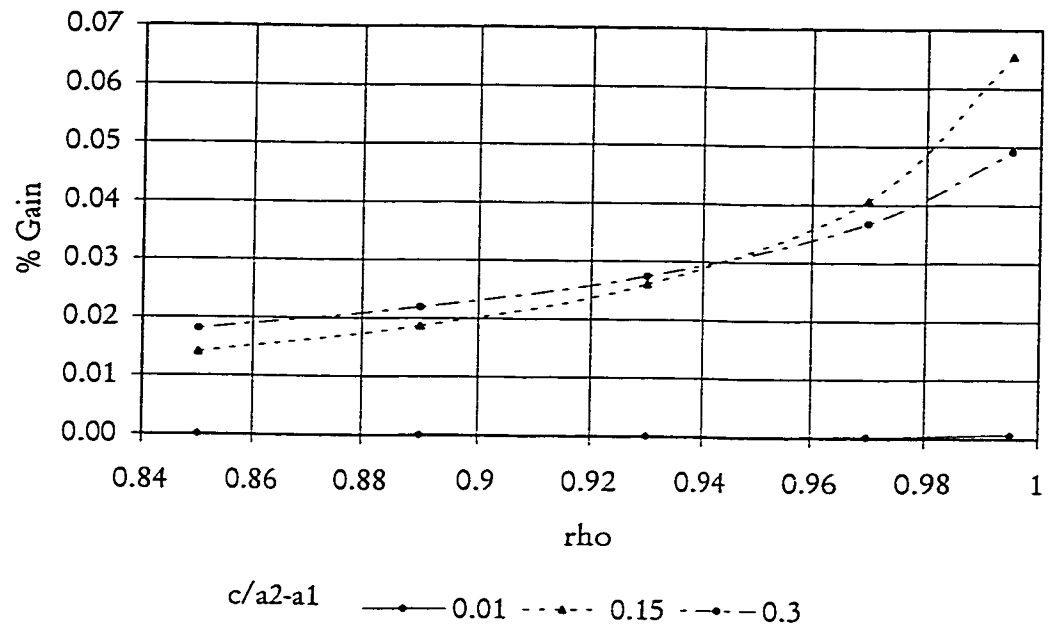


Figure 6-15: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 4, sigma = 1.75

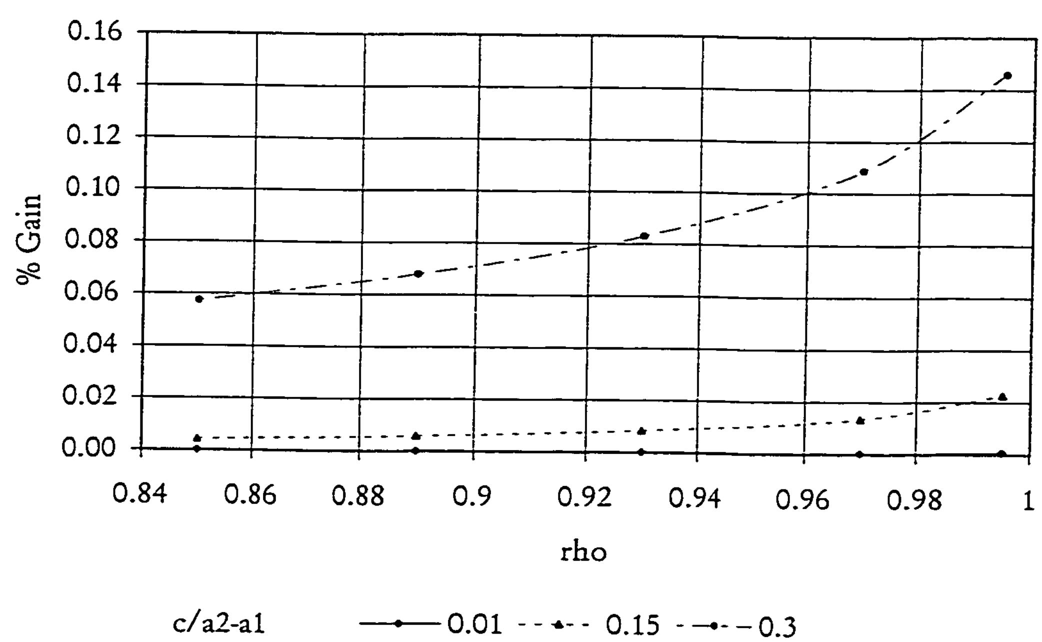


Figure 6-16: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 8, sigma = 1.75

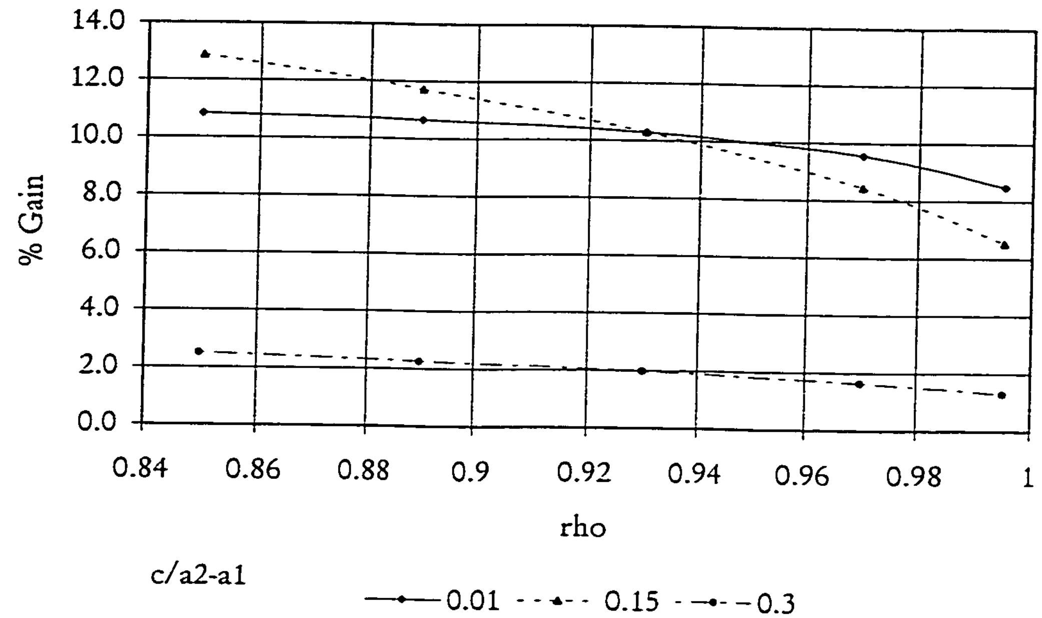


Figure 6-17: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 4, sigma = 1.25

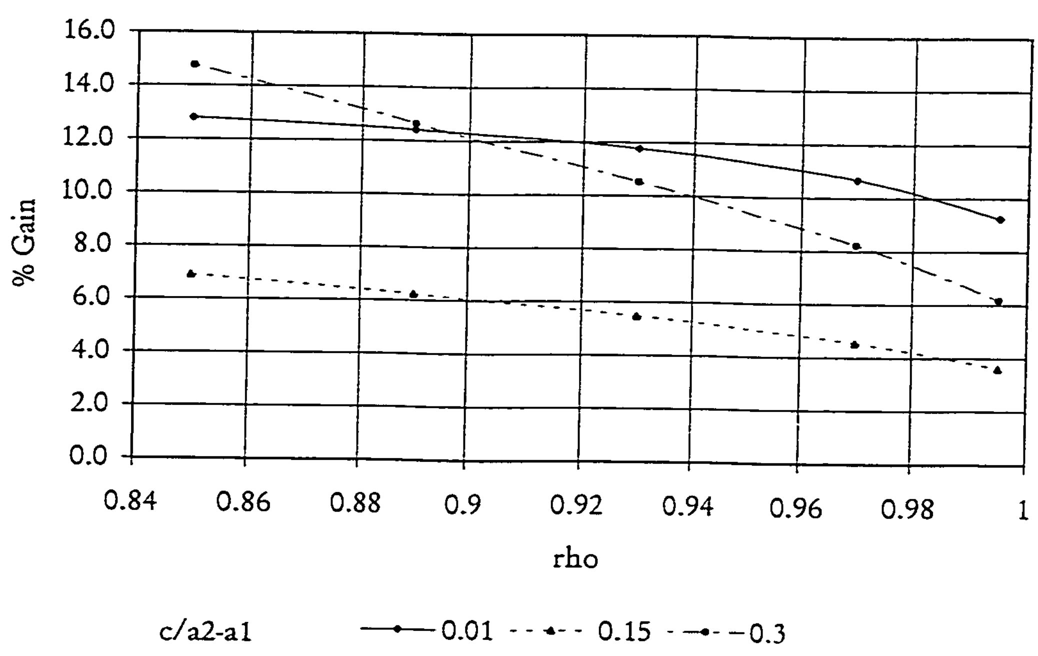


Figure 6-18: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 8, sigma = 1.25

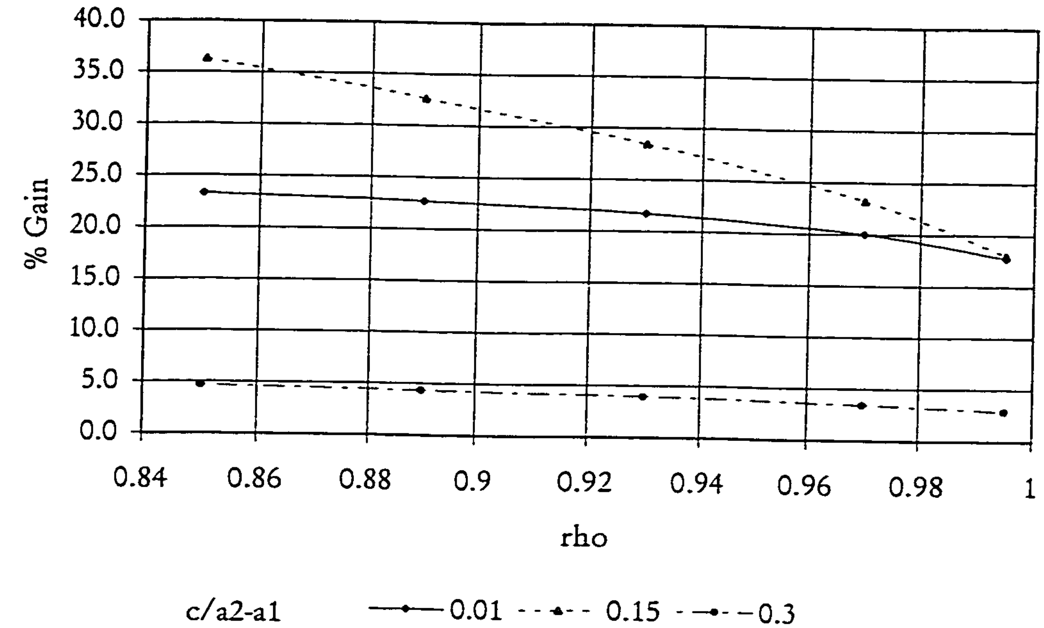


Figure 6-19: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 4, sigma = 1.75

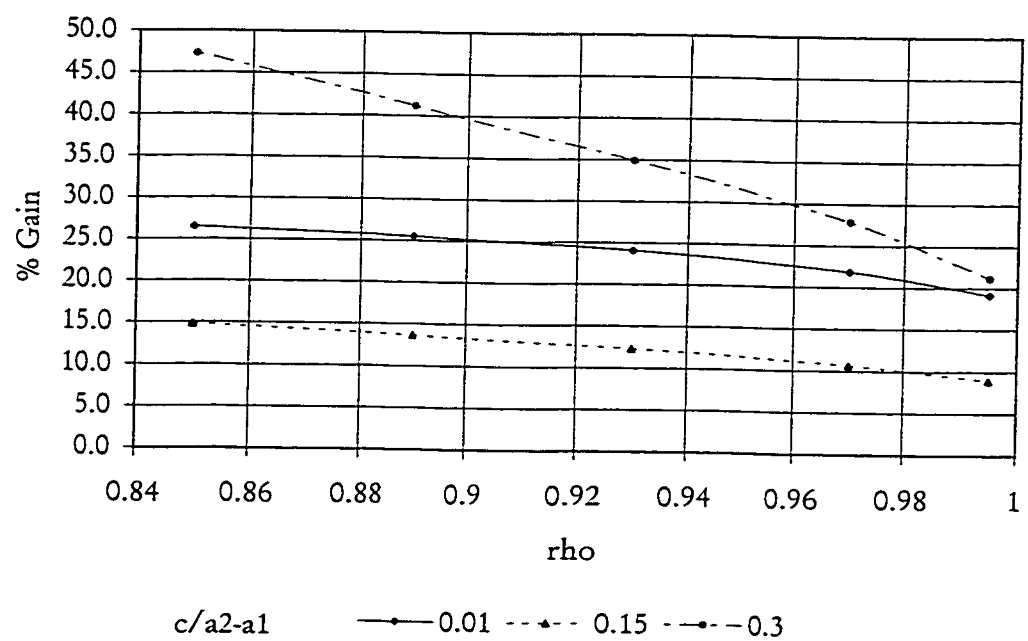


Figure 6-20: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 8, sigma = 1.75

get is set at L_1, use of Model 4 special case II will benefit a lot. The effect is also previously discussed in section 6.2.3.

6.3.4 MODEL 3 VS MODEL 4

In this section, model 3 and model 4 will be compared i.e., both the models with uniformity penalty & Model 3 without measurement error is considered. The level of K is again taken at 0.05.
In order to find the %gain if Model 4 is used instead of Model 3, the optimal mean obtain by Model 3 is substituted in Model 4. The cut off values are taken to be at the specification limits i.e.,

$$w_1=L_1,\ w_2=L_2$$

Each special case is compared with its respective case. Table 6-6 contains the results obtained from comparison between special cases of both the models i.e., target set at optimal mean of the process. The parameters taken are same as in chapter 4 and 5 i.e., $\frac{c}{a_1-a_2}, \frac{a_2-r}{a_1-a_2}$ & σ_y. The levels are same as used in chapter 5.

The results are shown graphically in figures 6-21 to 6-24. The results show significant %gain, even at low levels of error showing a %gain of about 5%, if model 4 is used. At higher level of measurement, the %gain in profit increases sharply. The reason is, as discussed in section 6.3.1., is the cut off values that play a vital role in reducing the effect of error. This effect is increased with the price and the cost as mean gets closer to L_1 with increase in cost the effect of misclassification increases and as a result, the % gain increases.

TABLE 6-6: Expected % gain in profit at different parameter settings. Model 3 Vs Model 4 (Special case I)

			$\frac{c}{a_1 - a_2}$		
σ_y	$\frac{a_2 - r}{a_1 - a_1}$	ρ	0.01	0.15	0.3
1.25	4	0.85	6.2990	21.7983	9.9898
		0.89	5.5626	18.4338	8.4155
		0.93	4.7484	14.8081	6.6948
		0.97	3.6994	10.3498	4.5442
		0.995	2.5104	5.6269	2.2170
	6	0.85	6.5249	11.5614	692.0175
		0.89	5.7426	9.8900	843.3492
		0.93	4.8694	8.0539	1523.9817
		0.97	3.7324	5.7253	1281.7686
		0.995	2.4315	3.1530	171.9031
	8	0.85	6.6157	10.1270	34.4850
		0.89	5.8130	8.6894	28.9684
		0.93	4.9143	7.1007	22.9607
		0.97	3.7406	5.0680	15.5010
		0.995	2.3950	2.7991	7.5800
1.75	4	0.85	9.4289	34.5902	11.8310
		0.89	8.3946	29.5609	10.1319
		0.93	7.2208	23.9681	8.1887
		0.97	5.6817	16.9329	5.6563
		0.995	3.9631	9.6476	2.9377
	6	0.85	9.4202	16.5437	165.6635
		0.89	8.3652	14.3283	152.4398
		0.93	7.1615	11.8210	136.2232
		0.97	5.5733	8.5700	111.6178
		0.995	3.7902	5.0448	75.9676
	8	0.85	9.4192	14.2311	55.2396
		0.89	8.3556	12.3659	46.7726
		0.93	7.1399	10.2454	37.3373
		0.97	5.5326	7.4760	25.4834
		0.995	3.7252	4.4412	13.3226

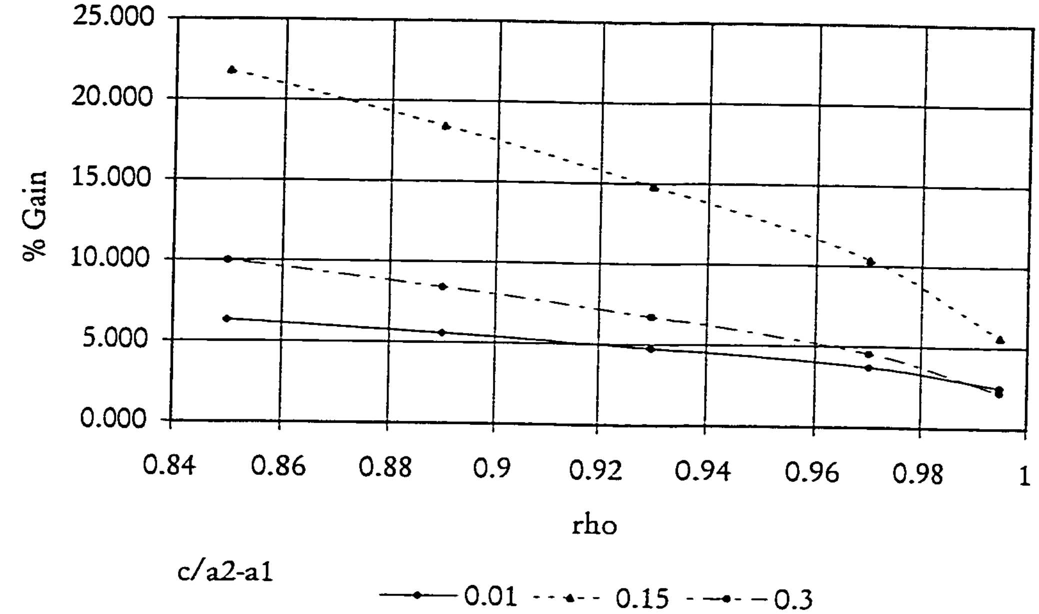


Figure 6-21: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 4, sigma = 1.25

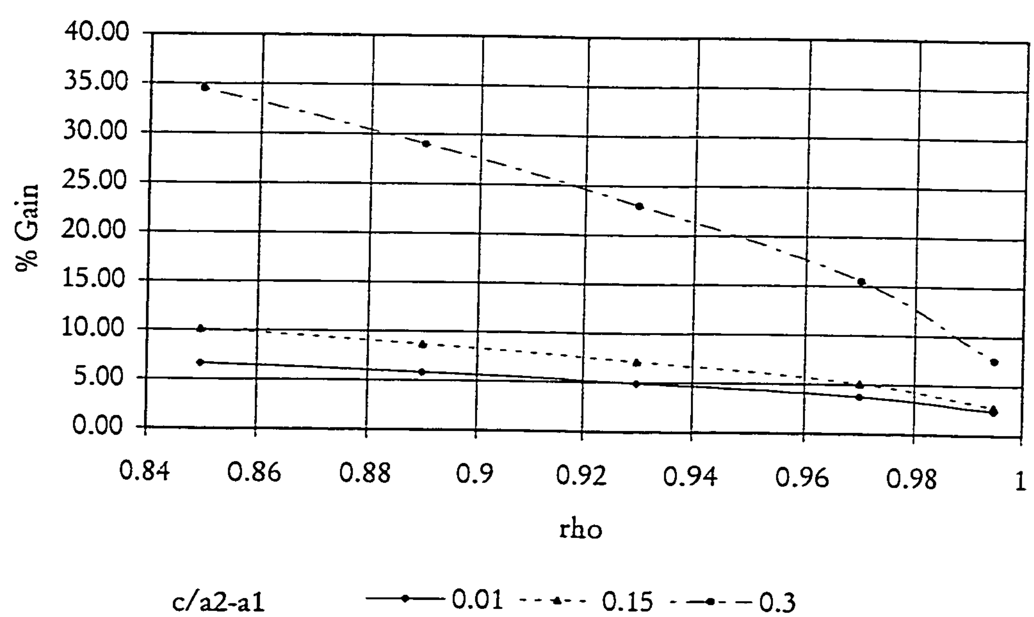


Figure 6-22: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 8, sigma = 1.25

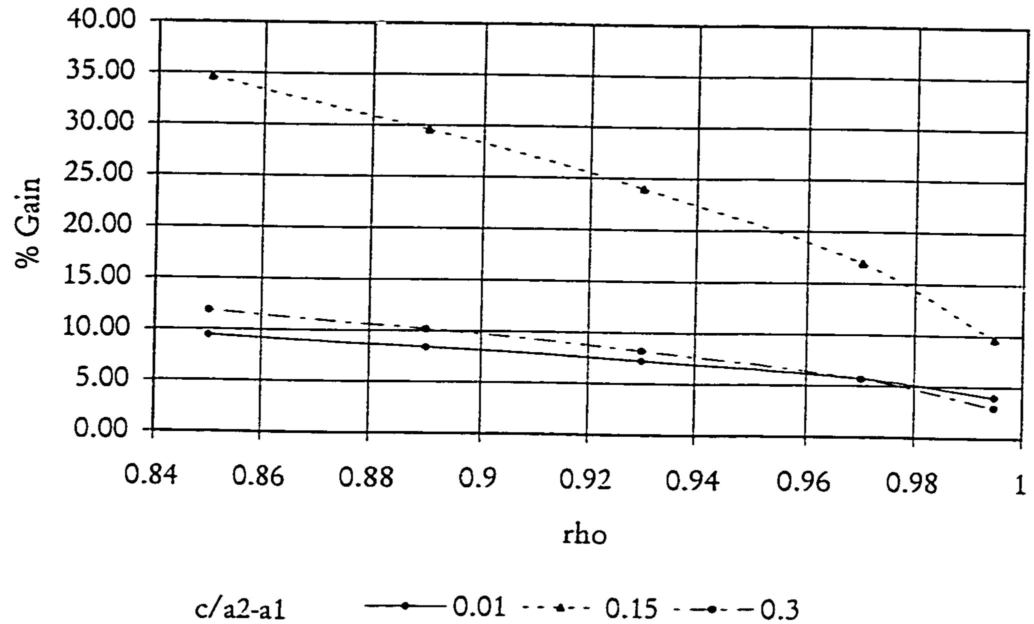


Figure 6-23: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 4, sigma = 1.75

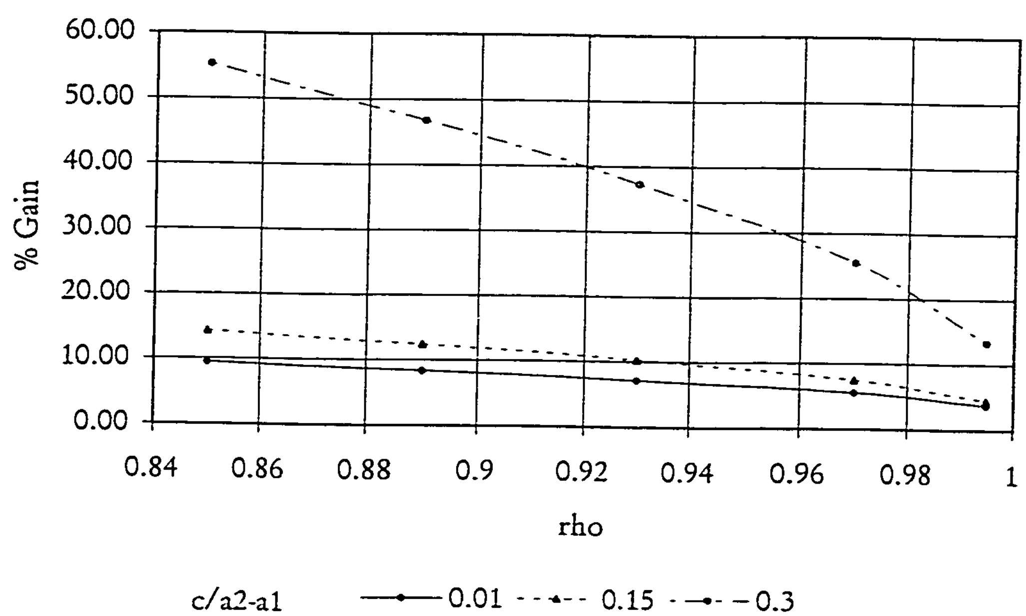


Figure 6-24: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 8, sigma = 1.75

For special case II, the results are presented in Table 6-7. The percentage gain in profit is significant even at low level of 'K'. The %gain is higher with the increase in measurement error which is what is expected, as seen in previous models and analytical analysis in section 6.2.4. The effect of cost and prices are similar in nature as in special case I. The results are shown graphically from figures 6-25 to 6-28

6.4 CONCLUSION

In this chapter, analytical and numerical comparison of the models is shown. The results show that the %gain in some case become very high even at low level of corresponding parameter settings. The analytic comparison shows the relationship between the models and it was shown that some models could be derived as a special case from some of the other generalized models.

TABLE 6-7: Expected % gain in profit at different parameter settings. Model 3 Vs Model 4 (Special case II)

σ_v	$\frac{a_2 - r}{a_1 - a_1}$	ρ	$\frac{c}{a_1 - a_2}$ 0.01	0.15	0.3
1.25	4	0.85	2.124136	12.93254	6.22629
		0.89	1.72640	10.38348	5.18029
		0.93	1.33889	7.84733	4.05392
		0.97	0.89212	4.97035	2.66366
		0.995	0.39653	2.05888	1.13898
	6	0.85	2.77077	6.95628	109.72139
		0.89	2.31024	5.76120	105.29858
		0.93	1.82940	4.49925	101.21034
		0.97	1.23213	2.95279	97.11539
		0.995	0.54275	1.25653	89.39634
	8	0.85	3.08814	6.29925	28.71731
		0.89	2.58720	5.24524	23.30934
		0.93	2.05441	4.11758	17.69447
		0.97	1.38256	2.71485	11.10805
		0.995	0.60542	1.15758	4.47158
1.75	4	0.85	3.01448	21.74071	6.41334
		0.89	2.43902	17.13482	5.36729
		0.93	1.86127	12.56175	4.20528
		0.97	1.19090	7.50549	2.72075
		0.995	0.50333	2.89898	1.13535
	6	0.85	3.74847	9.39533	49.09882
		0.89	3.09009	7.70455	43.64152
		0.93	2.39639	5.90869	37.17072
		0.97	1.54992	3.73523	27.61759
		0.995	0.65216	1.52050	14.02881
	8	0.85	4.08094	8.22147	51.85174
		0.89	3.37723	6.77920	40.82915
		0.93	2.62668	5.23070	29.73184
		0.97	1.70071	3.33104	17.44542
		0.995	0.71340	1.36394	6.50665

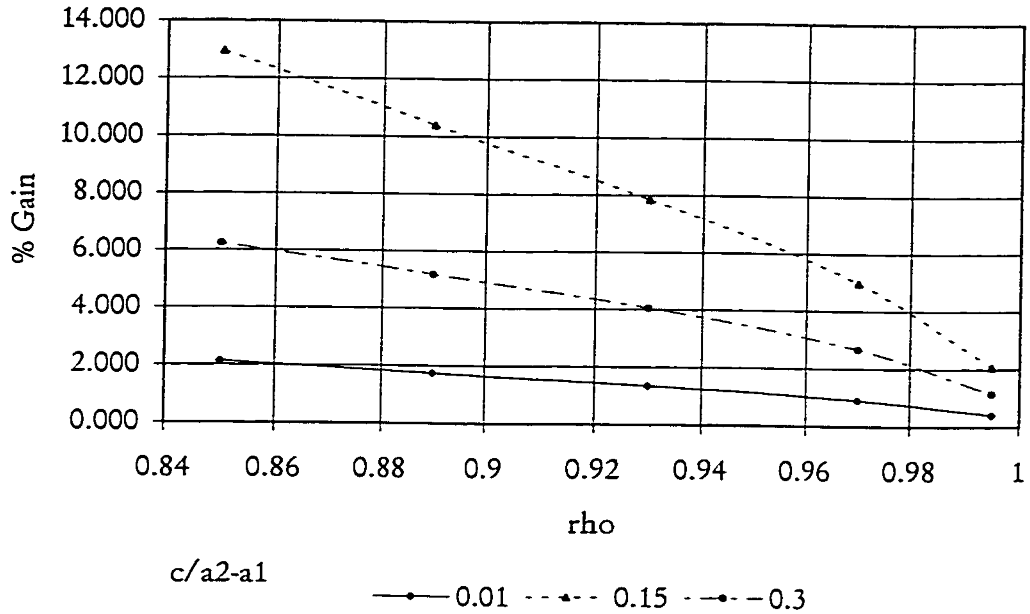


Figure 6-25: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 4, sigma = 1.25

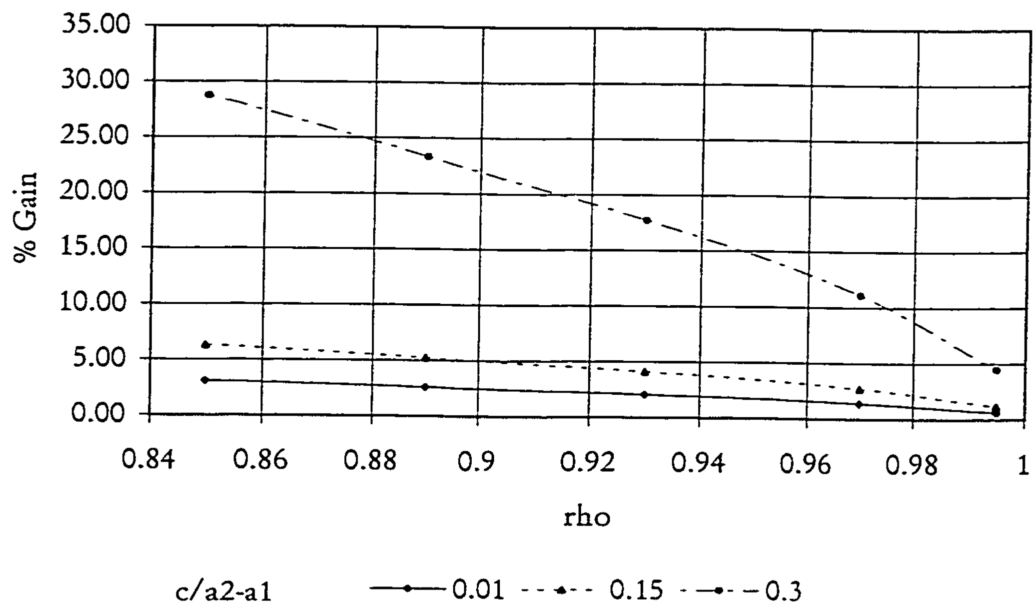


Figure 6-26: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 8, sigma = 1.25

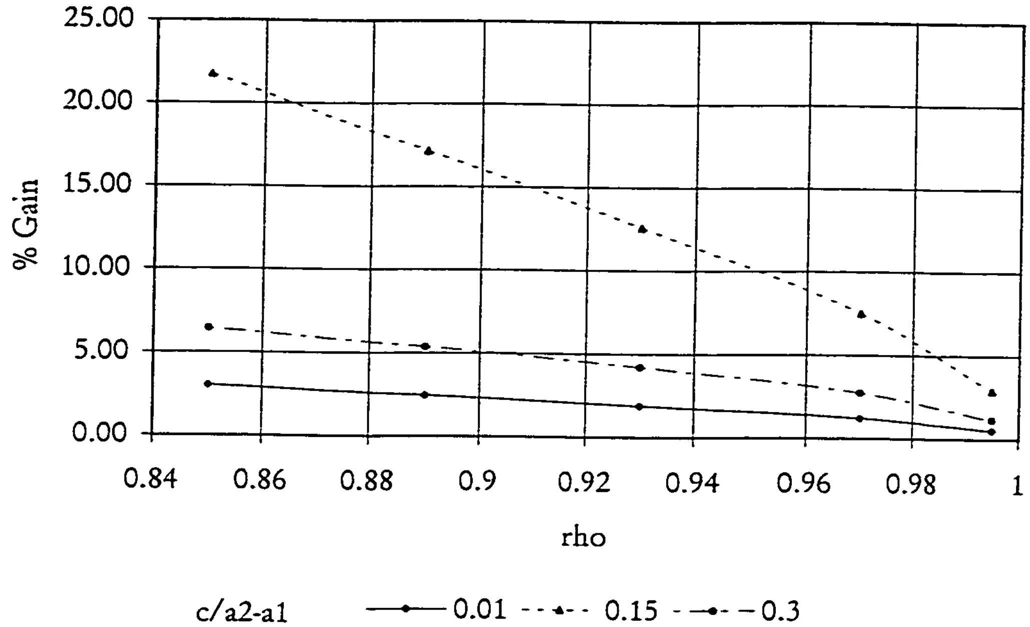


Figure 6-27: % Gain in expected profit versus measurment error (rho) at a2-r/a1-a2 = 4, sigma = 1.75

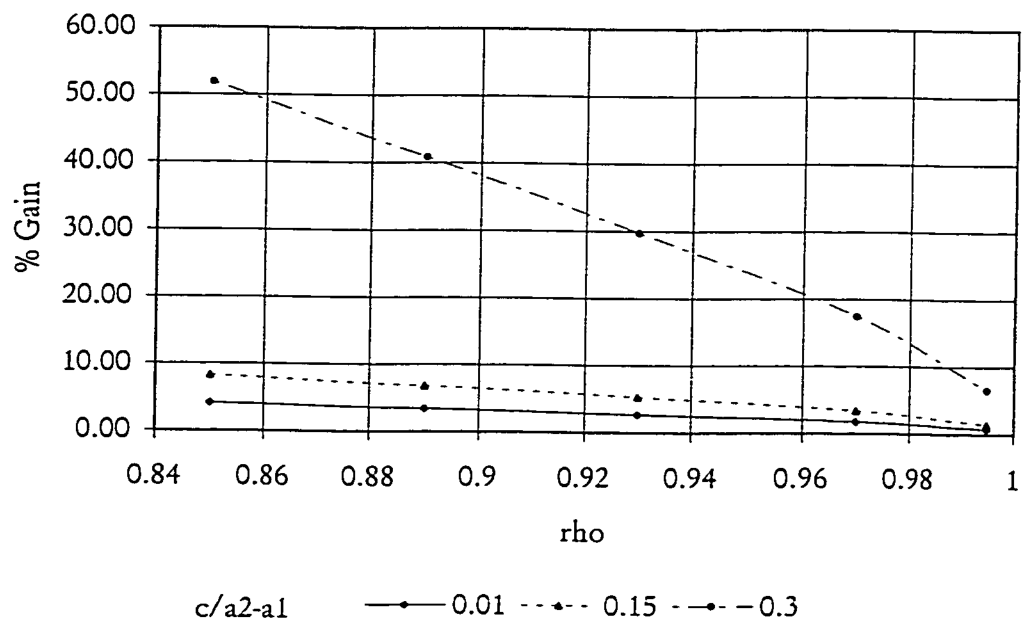


Figure 6-28: % Gain per a1 versus measurment error (rho) at a2-r/a1-a2 = 8, sigma = 1.75

CHAPTER 7

CONCLUSION

7.1 SUMMARY

The objective of this research is to develop different 'Process Targeting' models for single stage production processes with multi-class screening.

The, literature in the area was reviewed along with the Min et al. (1997) model in chapter 2. The first model that extends the Min et al. (1997) has been presented in chapter 3. In this chapter, the unrealistic assumption of error free measurement system was relaxed and a model for the case of unbiased and normally distributed measurement error was developed. Sensitivity analysis for this model was also discussed in the chapter. The effect of inconsistencies of the product was introduced, for the Min et al (1997) model, in chapter 4, followed by the two special cases in the same chapter. Sensitivity analysis was also presented in chapter 4 for these special cases. In chapter 5, model 2 and model

3, developed in chapter 3 and 4 are integrated. The two special cases are presented in this chapter of model 4 followed by its sensitivity analysis.

In chapter 6, all the four models are compared analytically and numerically. This chapter gives a good idea of when a simpler model can be approximated for the other, more generalized, model.

As the models presented in preceding chapters, are for three-class screening, a generalized form of the models i.e., for models with n-class screening is presented in section 7.2. In Section 7.4, several future directions of research are indicated. Lastly, in section 7.4, thesis conclusion is presented. The contributions form this thesis work can be summarized as follows:

- Three models were developed for various multi-class screening targeting problems
- The first extension, for the case of measurement systems with error performed well in reducing the effect of error of the profit.
- In the second extension, a penalty similar to that of "Taguchi Quadratic Loss Function" was introduced in the Min et al (1997) model. Two special cases were derived i.e., in the first case the target is assumed set at the process mean itself, while in the second special case the target is set the specification limit of the grade 1.
- In the third extension, the two models are integrated i.e., for the case of measurement error and uniformity penalty.

- The models are also compared with each other, both analytically and numerically. It was verified that the simpler models are special cases of the more generalized models.
- The numerical comparison of the model 2 and model 1 showed that the model 1 can be approximated for model 2 if the error is very small as well as the cost of manufacturing.

7.2 FUTURE RESEARCH

The work in this thesis can be extended in several directions. Some of these directions are:

- To generalize the above model to the case of n-class screening situation.
- To develop the models with other sampling plans.
- To extend the model to the situation where the process deteriorates. In the presented models, the process is assumed to be static and no deterioration with time is considered. Another extension is, to develop models for Processes with drift.
- TO generalize the model to multi-stage production or processes in series. One of the assumptions is that the production of the items is considered a one step process. The models can be extended for the case of machines in series.
- To extend the models where there is constraint for meeting certain Demand.
- To extend the model by integrating production decisions. This is a new area of research.

APPENDICES

APPENDIX A

NOMENCLATURE

Y	=	Quality characteristic to be measured
X	=	Observed value of 'Y'
a_1	=	Selling price of grade 1
a_2	=	Selling price of grade 2
r	=	Selling price of scrap
		Such that $a_1 > a_2 > r$
c_0	=	Fixed cost
c	=	Per unit cost
c_i	=	Inspection cost
w_1	=	Cut off value of 'X' for grade 1
w_2	=	Cut off value of 'X' for grade 2
b_{21}	=	Penalty associated with misclassification of grade 2 as grade 1 due to measurement error
b_{s1}	=	Penalty associated with misclassification of scrap as grade 1 due to measurement error.
b_{s2}	=	Penalty associated with misclassification of scrap as

grade 2 due to measurement error.

t = Target value of the process

K = Uniformity penalty

σ = Standard deviation of the process

σ_e = Standard deviation of the measurement error

L_1 = Lower specification limit for grade 1

L_2 = Lower specification limit for grade 2

APPENDIX B

PROGRAMS

PROGRAM FOR MODEL 1 (MIN et al. (1997)

```
In[4]:= (* C:\My Documents\atique\data\epml   -   1-2 *)

In[15]:= Clear [s, Φ, Φ1, Φ2, Φ3, μ, η, χ, σ]

In[16]:= a1 = 5.5; a2 = 5.07; r = 2.5; c0 = 0.1; ci = 0.04;

In[17]:= L1 = 41.5; L2 = 40;
```

$$\text{In[18]:=}\quad \eta = \frac{L1-\mu}{\sigma};\ \chi = \frac{L2-\mu}{\sigma};$$

$$\text{In[19]:=}\quad \Phi = \int_{-\infty}^{s} \frac{1}{\sqrt{2\pi}}\, e^{-\frac{z^2}{2}}\, dz;$$

```
In[20]:= s = -η; Φ1 = Φ;
         s = η; Φ2 = Φ;
         s = χ; Φ3 = Φ;

In[23]:= EPm1 = a1 * Φ1 + a2 * Φ2 + (r - a2) * Φ3 - c0 - ci - c (L1 - η * σ);

In[27]:= strm = OpenWrite["C:\My Documents\atique\data\data1.txt"];
         Off[NIntegrate::precw];
         Off[NIntegrate::nintp];
```

```
In[28]:= tp = {}; σlist = {1.25}; cls = {.0043};
         Do [
               Do[
                 tp = Append[tp, Timing[
                   -FindMinimum[Evaluate[-EPm1 /. {σ → σlist[[i]], c → cls[[j]]}],
                     {μ, 41}, MaxIterations → 100]]]
                , {i, 1}], {j, 1}];
             * tp; *
           Write[strm, tp];
         Close[strm];

In[371]:= On[NIntegrate::precw];
          On[NIntegrate::nintp];

In[31]:= tp
```

APPENDIX C

PROGRAMS

PROGRAM FOR MODEL 2 (EPM2)

```
<< Statistics`MultinormalDistribution`;

Clear[Φ, ζ, Φ1, Φ2, Φ3, Φ4, Φ5, Φ6, μ, η, χ, ρ, EPm2];
Clear[q1, q2, L1, L2, p1, p2, b1, b2, t1, t2, w1, w2, K1, K2, e];
```

$$\zeta = \int_{b1}^{b2} \frac{e^{-\frac{p^2}{2}}}{\sqrt{2\pi}}\, dp;$$

```
a1 = 5.5 ; r = 2.5 ; c0 = 0.1 ; ci = 0.04 ; b21 = 5 ; bs1 = 5.5 ; bs2 = 5 ;

L1 = 41.5; L2 = 40;

(z = {{1, ρ}, {ρ, 1}};
  ndist = MultinormalDistribution[{0, 0}, z]);

Φ = (CDF[ndist, {p1, p2}]) - (CDF[ndist, {q1, q2}]);
```

$$\eta = \frac{L1-\mu}{\sigma};\ \chi = \frac{L2-\mu}{\sigma};\ \delta_1 = \frac{w1-\mu}{\frac{\sigma}{\rho}};\ \delta_2 = \frac{w2-\mu}{\frac{\sigma}{\rho}};$$

```
b1 = δ1; b2 = ∞; Φ1 = ζ;
b1 = δ2; b2 = δ1; Φ2 = ζ;
b1 = -∞; b2 = δ2; Φ3 = ζ;
p1 = η; p2 = ∞; q1 = χ; q2 = ∞;
Φ4 = Φ - CDF[ndist, {η, δ1}] + CDF[ndist, {χ, δ1}];
p1 = χ; p2 = ∞; q1 = χ    ; q2 = δ1; Φ5 = Φ;
p1 = χ; p2 = δ1; q1 = χ; q2 = δ2; Φ6 = Φ;
```

$$\varphi = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{z^2}{2}};$$

```
EPm2 = a1 * Ψ1 + a2 * Ψ2 + r * Ψ3 - b21 * Ψ4 -
   bs1 * Ψ5 - bs2 * Ψ6 - c0 - c1 - rat (a1 - a2) * (L1 - η * σ);

EPm2 /. {μ → 43, σ → 1.5, ρ → .85, w1 → 41, w2 → 38, rat → .3, a2 → 5.00}

strm = OpenWrite["C:\My Documents\atique\data\epm2\data.txt"];
Off[NIntegrate::precw];
Off[NIntegrate::nintp];

Timing[tp = {}; a2lst = {4.9, 5.1, 5.17}; ratls = {0.01, 0.15, 0.3};
 σlist = {1.25, 1.75}; ρrr = {.85, .89, .93, .97, .995};

 Do[
       Do[
        Do[
         Do[
             tp = Append[tp, Timing[-FindMinimum[Evaluate[-EPm2 /.
                 {ρ → ρrr[[i]], σ → σlist[[q]],
                  a2 → a2lst[[p]], rat → ratls[[j]]}],
                 {μ, 42.8, 43}, {w1, 41.01, 41.05}, {w2, 40.01, 40.05},
               MaxIterations → 100]]]
          , {i, 5}],
         {p, 3}],
        {j, 3}],
       {q, 2}];

    * tp; *
  Write[strm, tp];

 Close[strm];]

On[NIntegrate::precw];
On[NIntegrate::nintp];
```

APPENDIX D

PROGRAMS

PROGRAM FOR MODEL 2 (EPM3)

In[1]:= Clear [s, Φ, Φ1, Φ2, Φ3, μ, η, χ, q1, q2, K1, σ]

In[3]:= a1 = 5.5; r = 2.5; c_0 = 0.1; c_i = 0.04;

In[4]:= L1 = 41.5; L2 = 40;
K1 = 0.05;

In[6]:= $\eta = \dfrac{L1 - \mu}{\sigma}$; $\chi = \dfrac{L2 - \mu}{\sigma}$;

In[7]:= $\Phi = \int_{-\infty}^{s} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$; $\varphi = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$;

In[8]:= s = $-\eta$; Φ1 = Φ;
s = η; Φ2 = Φ;
s = χ; Φ3 = Φ;

In[11]:= EPm1 = a1 * Φ1 + a2 * Φ2 + (r - a2) * Φ3 - c_0 -
c_i - rat (a1 - a2) * (L1 - η * σ) - K1 $\int_{\chi}^{\infty} (\mu + \sigma z - L1)^2 \varphi\, dz$;
EPm3 =
EPm1;

In[13]:=
```
strm = OpenWrite["C:\My Documents\atique\data\epm3\data-L1-2.txt"];
Off[NIntegrate::precw];
Off[NIntegrate::nintp];
```

```
In[16]:= tp = {}; a2lst = {4.9, 5.00, 5.04, 5.07, 5.10, 5.13, 5.15, 5.17};
         ratls = {.2, .25, .3};
         σlist = {1.5, 1.75, 2, 2.25};
         Do[
               Do[
                  Do[
                    tp = Append[tp, -FindMinimum[Evaluate[-EPm3 /.
                         {σ → σlist[[i]], a2 → a2lst[[j]], rat → ratls[[p]]}]
                      , {μ, 43}, MaxIterations → 100]]

                 , {j, 8}]
                        , {p, 3}]
                                 , {i, 4}];
             * tp; *
           Write[strm, tp];
         Close[strm];

In[57]:= On[NIntegrate::precw];
         On[NIntegrate::nintp];
```

APPENDIX E

PROGRAMS

PROGRAM FOR MODEL 2 (EPM4)

```
In[1]:= << Statistics`MultinormalDistribution`;

In[2]:= Clear [Φ, ξ, Φ1, Φ2, Φ3, Φ4, Φ5, Φ6, μ, η, χ, ρ, EPm4];
        Clear [q1, q2, L1, L2, p1, p2, b1, b2, t1, t2, w1, w2, K1, K2, e];
```

In[4]:= $\xi = \int_{b1}^{b2} \frac{e^{-\frac{p^2}{2}}}{\sqrt{2\pi}}\, dp;$

In[6]:= $a1 = 5.5\ ;\ r = 2.5\ ;\ c_0 = 0.1\ ;\ c_i = 0.04\ ;\ b21 = 5\ ;\ bs1 = 5.5\ ;\ bs2 = 5\ ;$

```
In[7]:=
        L1 = 41.5; L2 = 40;
        K1 = .15 * (a1 - a2);

In[11]:= (z = {{1, ρ}, {ρ, 1}};
          ndist = MultinormalDistribution[{0, 0}, z]);

In[12]:= Φ = (CDF[ndist, {p1, p2}]) - (CDF[ndist, {q1, q2}]);
```

In[13]:= $\eta = \frac{L1 - \mu}{\sigma};\ \chi = \frac{L2 - \mu}{\sigma};\ \delta_1 = \frac{w1 - \mu}{\frac{\sigma}{\rho}};\ \delta_2 = \frac{w2 - \mu}{\frac{\sigma}{\rho}};$

In[14]:= $b1 = \delta_1;\ b2 = \infty;\ \Phi1 = \xi;$
$b1 = \delta_2;\ b2 = \delta_1;\ \Phi2 = \xi;$
$b1 = -\infty;\ b2 = \delta_2;\ \Phi3 = \xi;$
$p1 = \eta;\ p2 = \infty;\ q1 = \chi;\ q2 = \infty;$
$\Phi4 = \Phi - \mathrm{CDF}[ndist, \{\eta, \delta_1\}] + \mathrm{CDF}[ndist, \{\chi, \delta_1\}];$
$p1 = \chi;\ p2 = \infty;\ q1 = \chi\ ;\ q2 = \delta_1;\ \Phi5 = \Phi;$
$p1 = \chi;\ p2 = \delta_1;\ q1 = \chi;\ q2 = \delta_2;\ \Phi6 = \Phi;$

In[20]:= $\varphi = \frac{1}{\sqrt{2\pi}} e^{-\frac{f^2}{2}};$

In[21]:= $\text{EPm4} = \text{a1} * \Psi 1 + \text{a2} * \Psi 2 + \text{r} * \Psi 3 - \text{b21} * \Psi 4 - \text{bs1} * \Psi 5 - \text{bs2} * \Psi 6 - c_0 - c_i - \text{rat}\,(\text{a1} - \text{a2}) * (\text{L1} - \eta * \sigma) - \text{K1} * \int_x^{\infty} ((\mu + \sigma f - \text{L1})^2 * \varphi)\, df;$

In[22]:=
```
strm = OpenWrite["C:\My Documents\atique\data\epm4\data(L-.15).txt"];
Off[NIntegrate::precw];
Off[NIntegrate::nintp];
```

In[25]:=
```
Timing[tp = {}; a2lst = {4.9, 5.1, 5.17}; ratls = {0.01, 0.15, 0.3};
 σlist = {1.25, 1.75}; ρrr = {.85, .89, .93, .97, .995};

 Do[
      Do[
       Do[
        Do[
            tp = Append[tp, Timing[-FindMinimum[Evaluate[-EPm4 /.
                {ρ → ρrr[[i]], σ → σlist[[q]],
                 a2 → a2lst[[p]], rat → ratls[[j]]}],
                {μ, 42.8, 43}, {w1, 41.01, 41.05},
              {w2, 40.01, 40.05}, MaxIterations → 100]]]
          , {i, 5}],
         {p, 3}],
        {j, 3}],
       {q, 2}];

   * tp; *
 Write[strm, tp];

Close[strm];]
```

In[329]:=
```
On[NIntegrate::precw];
On[NIntegrate::nintp];
```

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Vita

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